Solutionbank D1
Edexcel AS and A Level Modular Mathematics

Exercise A, Question 1

Question:

a Implement this algorithm with the fractions

i $2\frac{1}{4}$

ii $1\frac{1}{3}$.

1 Write the fractions in the form $\frac{a}{b}$ and $\frac{c}{d}$.

2 Let $e = ad$.

3 Let $f = bc$.

4 Print ' answer is $\frac{e}{f}$ '.

b What does this algorithm do?

Solution:

a **1** $\frac{a}{b} = \frac{9}{4}$ $\frac{c}{d} = \frac{4}{3}$ $a = 9, b = 4, c = 4, d = 3$

2 $e = ad = 9 \times 3 = 27$

3 $f = bc = 4 \times 4 = 16$

4 answer is $\frac{27}{16}$

b It divides the first fraction by the second fraction

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Exercise A, Question 2

Question:

a Implement this algorithm.

1 Let $A = 1, n = 1$.

2 Print A.

3 $A = A + 2n + 1$.

4 Let $n = n + 1$.

5 If $n \leq 10$ go to 2.

6 Stop.

b What does this algorithm produce?

Solution:

a **1** $A = 1 \quad n = 1$
2 Print 1
3 $A = 1 + 2 \times 1 + 1 = 4$
4 $n = 2$
5 $2 \leq 10$ go to 2
2 Print 4.
3 $A = 4 + 2 \times 2 + 1 = 9$
4 $n = 3$
5 $3 \leq 10$ go to 2
2 Print 9
3 $A = 9 + 2 \times 3 + 1 = 16$
4 $n = 4$
5 $4 \leq 10$ go to 2
2 Print 16
3 $A = 16 + 2 \times 4 + 1 = 25$
4 $n = 5$
5 $5 \leq 10$ go to 2
2 Print 25
3 $A = 25 + 2 \times 5 + 1 = 36$
4 $n = 6$
5 $6 \leq 10$ go to 2
2 Print 36
3 $A = 36 + 2 \times 6 + 1 = 49$
4 $n = 7$
5 $7 \leq 10$ go to 2
2 Print 49
3 $A = 49 + 2 \times 7 + 1 = 64$
4 $n = 8$
5 $8 \leq 10$ go to 2
2 Print 64
3 $A = 64 + 2 \times 8 + 1 = 81$
4 $n = 9$
5 $9 \leq 10$ go to 2
2 Print 81
3 $A = 81 + 2 \times 9 + 1 = 100$
4 $n = 10$
5 $10 \leq 10$ go to 2
2 Print 100
3 $A = 100 + 2 \times 10 + 1 = 121$
4 $n = 11$
5 $11 \not\leq 10$
6 stop

Output 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

b It finds the squares of the first 10 natural numbers.

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Exercise A, Question 3

Question:

a Use a trace table to implement the following algorithm when

i $A = 253$ and $r = 12$,

ii $A = 79$ and $r = 10$,

iii $A = 4275$ and $r = 50$.

1 Input A, r.

2 Let $C = \frac{A}{r}$ to 3 decimal places.

3 If $|r - C| \leq 10^{-2}$ go to 7.

4 Let $s = \frac{1}{2}(r + C)$ to 3 decimal places.

5 Let $r = s$.

6 Go to 2.

7 Print r.

8 Stop.

b What does the algorithm produce?

> This requires you to use the modulus function. If $x \neq y$, $|x - y|$ is the positive difference between x and y. For example, $|3.2 - 7| = 3.8$.

Solution:

a i

Step	A	r	c	$\|r-c\|$	s	Print r
1	253	12				
2			21.083			
3				9.083		
4					16.542	
5		16.542				
$6 \to 2$			15.294			
3				1.248		
4					15.918	
5		15.918				
$6 \to 2$			15.894			
3				0.024		
4					15.906	
5		15.906				
$6 \to 2$			15.906			
$3 \to 7$				0		
7						$r = 15.906$
8 stop						

ii

Step	A	r	c	$\|r-c\|$	s	Print r
1	79	10				
2				7.9		
3				2.1		
4					8.95	
5		8.95				
$6 \to 2$			8.827			
3				0.123		
4					8.889	
5		8.889				
$6 \to 2$			8.887			
$3 \to 7$				0.002		
7						Print 8.889

iii

Step	A	r	c	$\lvert r-c \rvert$	s	Print r
1	4275	50				
2			85.5			
3				35.5		
4					67.75	
5		67.75				
$6 \to 2$			63.100			
3				4.65		
4					65.425	
5		65.425				
$6 \to 2$			65.342			
3				0.083		
4					65.384	
5		65.384				
$6 \to 2$			65.383			
$3 \to 7$				0.001		
7						Print 65.384

b Finds the square root of A.

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Exercise A, Question 4

Question:

Use the algorithm in Example 3 to evaluate

a 244×125

b 125×244

c 256×123.

Solution:

a

A	*B*
~~244~~	~~125~~
~~122~~	~~250~~
61	500
~~30~~	~~1000~~
15	2000
7	4000
3	8000
1	16 000
Total	30 500

b

A	*B*
125	244
~~62~~	~~488~~
31	976
15	1952
7	3904
3	7808
1	15 616
Total	30 500

c

A	*B*
~~256~~	~~123~~
~~128~~	~~246~~
~~64~~	~~492~~
~~32~~	~~984~~
~~16~~	~~1968~~
~~8~~	~~3936~~
~~4~~	~~7872~~
~~2~~	~~15744~~
1	31 488
Total	31 488

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Exercise B, Question 1

Question:

Apply the flow chart in Example 5 to the following equations.

a $4x^2 - 12x + 9 = 0$

b $-6x^2 + 13x + 5 = 0$

c $3x^2 - 8x + 11 = 0$

Solution:

a

a	b	c	d	$d < 0$?	$d = 0$?	x
4	−12	9	0	No	Yes	1.5

Equal roots $x = 1.5$

b

a	b	c	d	$d < 0$?	$d = 0$?	x_1	x_2
−6	13	5	289	No	No	$-\frac{1}{3}$	$\frac{5}{2}$

Roots are $-\frac{1}{3}$ and $\frac{5}{2}$

c

a	b	c	d	$d < 0$?
3	−8	11	−68	Yes

No real roots

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Exercise B, Question 2

Question:

a Apply the flow chart in Example 6 to the following data.

i $u_1 = 28, u_2 = 26, u_3 = 23, u_4 = 25, u_5 = 21$

ii $u_1 = 11, u_2 = 8, u_3 = 9, u_4 = 8, u_5 = 5$

b If box 4 is altered to Is $T > A$?, how will this affect the output?

c Which box would need to be altered if the algorithm had to be applied to a list of 8 numbers?

Solution:

a i

	n	A	T	$T < A$?	$n < 5$?
box 1	1	28			
box 2	2				
box 3			26		
box 4				Yes	
box 5		26			
box 6					Yes
box 2	3				
box 3			23		
box 4				Yes	
box 5		23			
box 6					Yes
box 2	4				
box 3			25		
box 4				No	
box 6					Yes
box 2	5				
box 3			21		
box 4				Yes	
box 5		21			
box 6					No
box 7	output is 21				

ii

	n	A	T	$T < A$?	$n < 5$?
box 1	1	11			
box 2	2				
box 3			8		
box 4				Yes	
box 5		8			
box 6					Yes
box 2	3				
box 3			9		
box 4				No	
box 6					Yes
box 2	4				
box 3			8		
box 4				No	
box 6					Yes
box 2	5				
box 3			5		
box 4				Yes	
box 5		5			
box 6					No
box 7	output is 5				

b It will find the largest number in the list.

c box 6 – changed to 'Is $n < 8$?'

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Exercise B, Question 3

Question:

Euclid's algorithm is applied to two non-zero integers a and b.

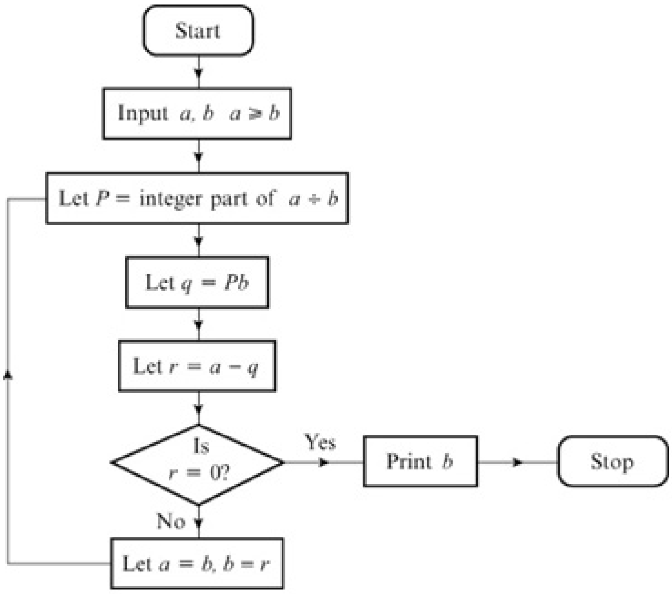
a Apply Euclid's algorithm to

i 507, 52

ii 884, 85

iii 4845, 3795

b What does the algorithm do?



Solution:

a i

a	*b*	*p*	*q*	*r*	*r* = 0 ?
507	52				
		9			
			468		
				39	
					No
52	39				
		1			
			39		
				13	
					No
39	13				
		3			
			39		
				0	
					Yes

Print 13

ii

a	*b*	*p*	*q*	*r*	*r* = 0 ?
884	85				
		10			
			850		
				34	
					No
85	34				
		2			
			68		
				17	
					No
34	17				
		2			
			34		
				0	
					Yes

Print 17

iii

a	*b*	*p*	*q*	*r*	*r* = 0 ?
4845	3795				
		1			
			3795		
				1050	
					No
3795	1050				
		3			
			3150		
				645	
					No
1050	645				
		1			
			645		
				405	
					No
645	405				
		1			
			405		
				240	
					No
405	240				
		1			
			240		
				165	
					No
240	165				
		1			
			165		
				75	
					No
165	75				
		2			
			150		
				15	
					No
75	15				
		5			
			75		
				0	
					Yes

Print 15

b Finds the HCF.

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Exercise B, Question 4

Question:

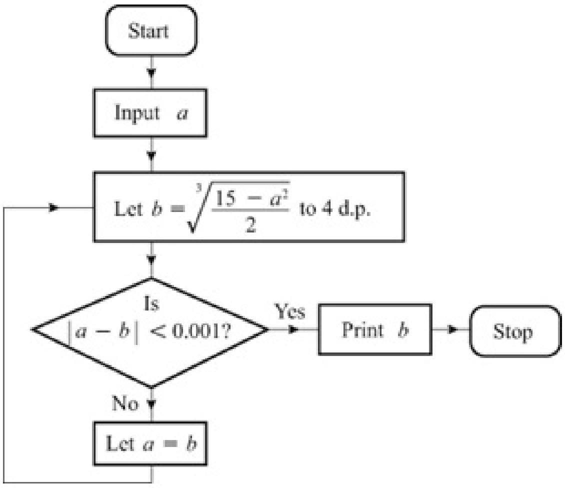
The equation $2x^3 + x^2 - 15 = 0$ may be solved by the iteration

$$x_{n+1} = \sqrt[3]{\frac{15 - x^2}{2}}$$

using the chart opposite.

a Use $a = 2$ to find a root of the equation.

b Use $a = 20$ to find a root of the equation. What do you notice?



Solution:

a

a	b	$\lvert a-b \rvert$	$\lvert a-b \rvert < 0.001$?
2	1.7652	0.2348	No
1.7652	1.8112	0.046	No
1.8112	1.8029	0.0083	No
1.8029	1.8044	0.0015	No
1.8044	1.8041	0.0003	Yes

output: 1.8041

b

a	b	$\lvert a-b \rvert$	$\lvert a-b \rvert < 0.001$?
20	−5.7740	25.774	No
−5.7740	−2.0931	3.6809	No
−2.0931	1.7446	3.8377	No
1.7446	1.8149	0.0703	No
1.8149	1.8022	0.0127	No
1.8022	1.8045	0.0023	No
1.8045	1.8041	0.0004	Yes

output 1.8041 – same root

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Exercise C, Question 1

Question:

Use the bubble sort to arrange the list

8 3 4 6 5 7 2

into

a ascending order,

b descending order.

Solution:

a

8 3 4 6 5 7 2

Bubbling left to right

1st pass	3	4	6	5	7	2	8
2nd pass	3	4	5	6	2	7	8
3rd pass	3	4	5	2	6	7	8
4th pass	3	4	2	5	6	7	8
5th pass	3	2	4	5	6	7	8
6th pass	2	3	4	5	6	7	8

sort complete

b Bubbling left to right

1st pass	8	4	6	5	7	3	2
2nd pass	8	6	5	7	4	3	2
3rd pass	8	6	7	5	4	3	2
4th pass	8	7	6	5	4	3	2

sort complete

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Exercise C, Question 2

Question:

Use a quick sort to arrange the list

22 17 25 30 11 18 20 14 7 29

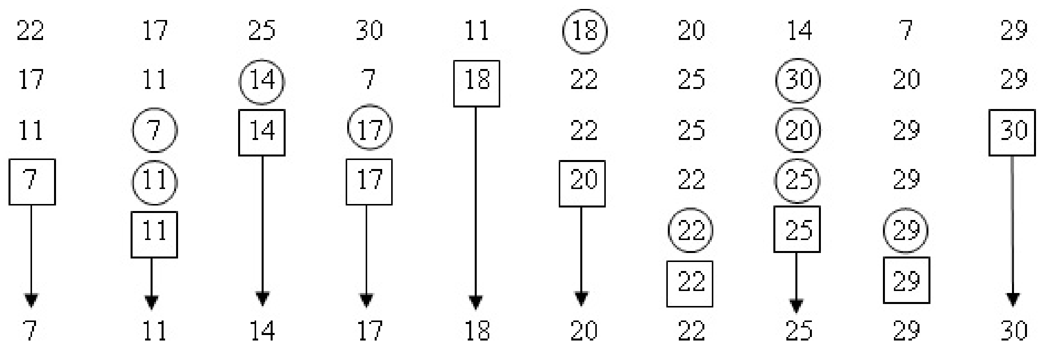
into

a ascending order,

b descending order.

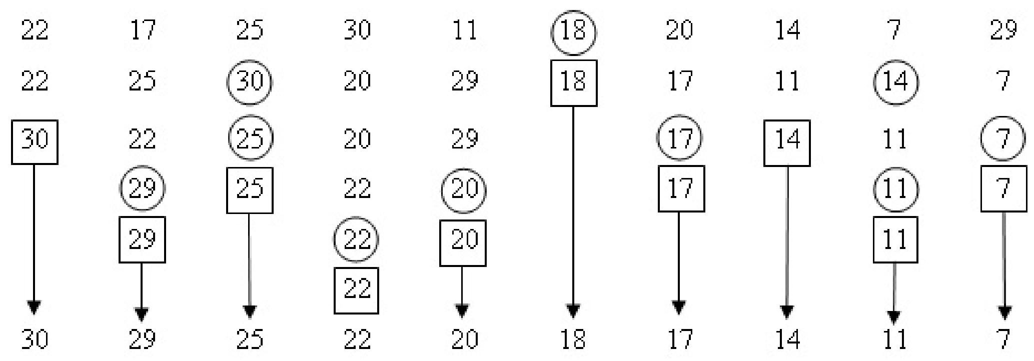
Solution:

a



sort complete

b



sort complete

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Exercise C, Question 3

Question:

Sort the letters below into alphabetical order using

a a bubble sort,

b a quick sort.

N H R K S C J E M P L

Solution:

a

N H R K S C J E M P L

Bubbling left to right

H	N	K	R	C	J	E	M	P	L	S
H	K	N	C	J	E	M	P	L	R	S
H	K	C	J	E	M	N	L	P	R	S
H	C	J	E	K	M	L	N	P	R	S
C	H	E	J	K	L	M	N	P	R	S
C	E	H	J	K	L	M	N	P	R	S

sort complete

b

N	H	R	K	S	C	J	E	M	P	L
$\boxed{C}$	N	H	R	K	S	J	E	M	P	L
	H	E	$\boxed{J}$	N	R	K	S	M	P	L
	$\boxed{E}$	H		N	R	K	M	P	L	$\boxed{S}$
		$\boxed{H}$		K	L	$\boxed{M}$	N	R	P	
				K	$\boxed{L}$		N	P	$\boxed{R}$	
				$\boxed{K}$			N	$\boxed{P}$		
							$\boxed{N}$			
C	E	H	J	K	L	M	N	P	R	S

sort complete

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Exercise C, Question 4

Question:

The list shows the test results of a group of students.

Alex	33	Hugo	9
Alison	56	Janelle	89
Amy	93	Josh	37
Annie	51	Lucy	57
Dom	77	Myles	19
Greg	91	Sam	29
Harry	49	Sophie	77

Produce a list of students, in descending order of their marks, using

a a bubble sort.

b a quick sort.

Solution:

a

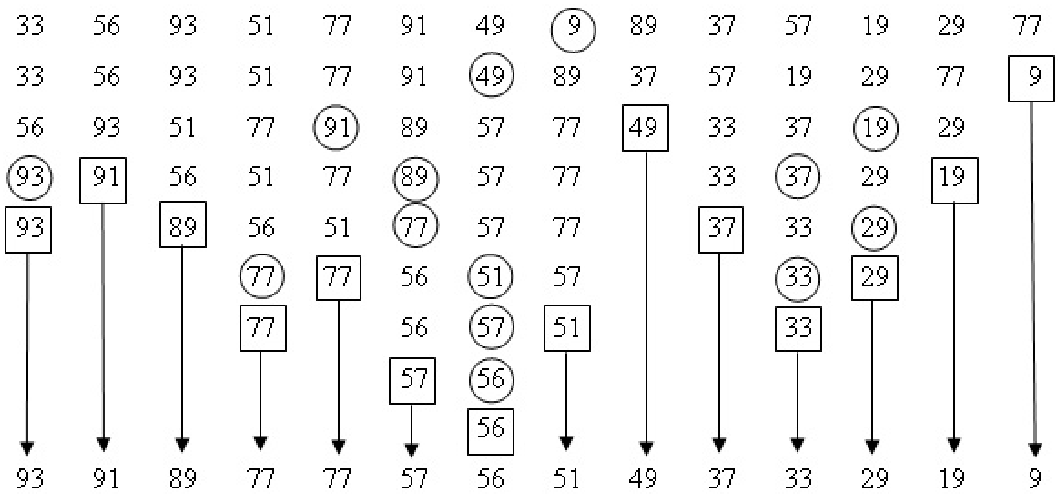
33	56	93	51	77	91	49	9	89	37	57	19	29	77
56	93	51	77	91	49	33	89	37	57	19	29	77	9
93	56	77	91	51	49	89	37	57	33	29	77	19	9
93	77	91	56	51	89	49	57	37	33	77	29	19	9
93	91	77	56	89	51	57	49	37	77	33	29	19	9
93	91	77	89	56	57	51	49	77	37	33	29	19	9
93	91	89	77	57	56	51	77	49	37	33	29	19	9
93	91	89	77	57	56	77	51	49	37	33	29	19	9
93	91	89	77	57	77	56	51	49	37	33	29	19	9
93	91	89	77	77	57	56	51	49	37	33	29	19	9

sort complete

So list is:

Amy	93	Annie	51
Greg	91	Harry	49
Janelle	89	Josh	37
Sophie	77	Alex	33
Dom		Sam	29
Lucy	57	Myles	19
Alison	56	Hugo	9

b



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Exercise C, Question 5

Question:

Sort the numbers listed below into ascending order using

a a bubble sort,

b a quick sort.

> This question gives you a good comparison between the efficiencies of the two algorithms.

453	330	405	792	516	162	465	870	431
927	129	348	34	107	64	253	382	411
147	389	597	414	620	425	73	275	212
482	302	52	868	144	65	471	930	766
243	578	274	630	281	732	114	517	322
748	517	492	331					

Solution:

The sort is left as an exercise for the reader.

The quick sort should be notably faster than the bubble sort.

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Exercise D, Question 1

Question:

Use the binary search algorithm to try to locate

a Connock,

b Walkey,

c Peabody.

in the list below.

1 Berry
2 Connock
3 Ladley
4 Sully
5 Tapner
6 Walkey
7 Ward
8 Wilson

Solution:

a 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Connock before Tapner reject $5 \to 8$

2nd pivot $\left[\frac{1+4}{2}\right]=3$ Ladley. Connock before Ladley reject $3 \to 4$

3rd pivot $\left[\frac{1+2}{2}\right]=2$ Connock. Target found at position 2.

b 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Walkey after Tapner reject $1 \to 5$

2nd pivot $\left[\frac{6+8}{2}\right]=7$ Ward. Walkey before Ward reject $7 \to 8$

3rd pivot 6 Walkey. Target found at position 6.

c 1st pivot $\left[\frac{1+8}{2}\right]=5$ Tapner. Peabody before Tapner reject $5 \to 8$

2nd pivot $\left[\frac{1+4}{2}\right]=3$ Ladley. Peabody after Ladley reject $1 \to 3$

3rd pivot 4 Sully. Sully is not Peabody reject 4.

List empty. Peabody not in list.

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Exercise D, Question 2

Question:

Use the binary search algorithm to try to locate

a 21,

b 5

in the list below.

1 3
2 4
3 7
4 9
5 10
6 13
7 15
8 17
9 18
10 20
11 21
12 24

Solution:

a 1st pivot $\left[\frac{1+12}{2}\right]=7$ (number 15) $21>15$ reject $1\to 7$

2nd pivot $\left[\frac{8+12}{2}\right]=10$ (number 20) $21>20$ reject $8\to 10$

3rd pivot $\left[\frac{11+12}{2}\right]=12$ (number 24) $21<24$ reject 12

4th pivot 11 (number 21). Target found at position 11.

b 1st pivot $\left[\frac{1+12}{2}\right]=7$ (number 15) $5<15$ reject $7\to 12$

2nd pivot $\left[\frac{1+6}{2}\right]=4$ (number 9) $5<9$ reject $4\to 6$

3rd pivot $\left[\frac{1+3}{2}\right]=2$ (number 4) $2<5$ reject $1\to 2$

4th pivot 3 (number 7) $5\neq 7$ reject 3

List empty. 5 not in list.

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Exercise D, Question 3

Question:

Use the binary search algorithm to try to locate

a Fredco,

b Matt,

c Elliot

in the list below.

1 Adam
2 Alex
3 Des
4 Doug
5 Ed
6 Emily
7 Fredco
8 George
9 Harry
10 Jess
11 Katie
12 Leo
13 Lottie
14 Louis
15 Matt
16 Miranda
17 Oli
18 Ramin
19 Saul
20 Simon

Solution:

a 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Fredco before Katie, reject $11 \rightarrow 20$

2nd pivot $\left[\frac{1+10}{2}\right]=6$ Emily. Fredco after Emily, reject $1 \rightarrow 6$

3rd pivot $\left[\frac{7+10}{2}\right]=9$ Harry. Fredco before Harry, reject $9 \rightarrow 10$

4th pivot $\left[\frac{7+8}{2}\right]=8$ George. Fredco before George, reject 8

5th pivot 7 Fredco Target found at position 7.

b 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Matt after Katie, reject $1 \rightarrow 11$

2nd pivot $\left[\frac{12+20}{2}\right]=16$ Miranda. Matt before Miranda, reject $16 \rightarrow 20$

3rd pivot $\left[\frac{12+15}{2}\right]=14$ Louis. Matt after Louis, reject $12 \rightarrow 14$

4th pivot 15 Matt. Target found at position 15.

c 1st pivot $\left[\frac{1+20}{2}\right]=11$ Katie. Elliot before Katie, reject $11 \rightarrow 20$

2nd pivot $\left[\frac{1+10}{2}\right]=6$ Emily. Elliot before Emily, reject $6 \rightarrow 10$

3rd pivot $\left[\frac{1+5}{2}\right]=3$ Des. Elliot after Des, reject $1 \rightarrow 3$

4th pivot $\left[\frac{4+5}{2}\right]=5$ Ed. Elliot after Ed, reject $4 \rightarrow 5$

List empty. Elliot not in list.

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Exercise D, Question 4

Question:

The 26 letters of the English alphabet are listed, in order.

a Apply the binary search algorithm to locate the letter P.

b What is the maximum number of iterations needed to locate any letter?

Solution:

a 1st pivot $\left[\frac{1+26}{2}\right] = 14$ N. P after N, reject $1 \to 14$

2nd pivot $\left[\frac{15+26}{2}\right] = 21$ U. P before U, reject $21 \to 26$

3rd pivot $\left[\frac{16+20}{2}\right] = 18$ R. P after R, reject $18 \to 20$

4th pivot $\left[\frac{16+17}{2}\right] = 17$ Q. P before Q, reject 17

5th pivot 16 P. Target found at position 16.

b After each pass the list halves.
After 1 pass at most 13 letters remain in the list.
After 2 passes at most 6 letters remain in the list.
After 3 passes at most 3 letters remain in the list.
After 4 passes at most 1 letter remains in the list.
After 5 passes the final letter is examined.
so at most 5 passes.

Alternative method

Each pass halves the list. So we use need to find the smallest value of n for which

$$\begin{aligned} 2^n &> 26 \\ n\log 2 &> \log 26 \\ n &> \frac{\log 26}{\log 2} \\ n &> 4.7 \\ \text{so } n &= 5. \end{aligned}$$

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Exercise D, Question 5

Question:

The binary search algorithm is applied to an ordered list of n items. Determine the maximum number of iterations needed when n is equal to

a 100

b 1000

c 10 000.

You may find it helpful to record the maximum length of the list after each iteration.

Solution:

a

Pass	1	2	3	4	5	6	7
Maximum number of items remaining in list	50	25	12	6	3	1	0

So 7 passes
alternative

$$\begin{aligned} 2^m &> 100 \\ \log 2^m &> \log 100 \\ m\log 2 &> \log 100 \\ m &> \frac{\log 100}{\log 2} \\ m &> 6.64 \text{ so 7 passes} \end{aligned}$$

b

Pass	1	2	3	4	5	6	7	8	9	10
Maximum number of items remainings in list	500	250	125	62	31	15	7	3	1	0

So 10 passes
alternative

$$\begin{aligned} 2^m &> 1000 \\ \log 2^m &> \log 1000 \\ m\log 2 &> \log 1000 \\ m &> \frac{\log 1000}{\log 2} \\ m &> 9.97 \text{ so 10 passes} \end{aligned}$$

c

$$\begin{aligned} 2^m &> 10\,000 \\ \log 2^m &> \log 10\,000 \\ m\log 2 &> \log 10\,000 \\ m &> \frac{\log 10\,000}{\log 2} \\ m &> 13.3 \text{ so 14 passes} \end{aligned}$$

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Exercise E, Question 1

Question:

18	4	23	8	27	19	3	26	30	35	32

The above items are to be packed in bins of size 50.

a Calculate the lower bound for the number of bins.

b Pack the items into the bins using

i the first-fit algorithm,

ii the first-fit decreasing algorithm,

iii the full-bin algorithm.

Solution:

a Lower bound $= \frac{18+4+23+8+27+19+3+26+30+35+32}{50} = \frac{225}{50}$

$= 4.5$

Therefore 5 bins (4 bins will be insufficient)

b **i** Bin 1: $18+4+23+3$
Bin 2: $8+27$
Bin 3: $19+26$
Bin 4: 30
Bin 5: 35
Bin 6: 32

ii Putting list into descending order

35 32 30 27 26 23 19 18 8 4 3

Bin 1: $35+8+4+3$
Bin 2: $32+18$
Bin 3: $30+19$
Bin 4: $27+23$
Bin 5: 26

iii For example

$$\left.\begin{array}{l}\text{Bin 1: } 32+18\\ \text{Bin 2: } 27+23\\ \text{Bin 3: } 35+8+4+3\end{array}\right\}\quad\text{Full bins}$$

Bin 4: $19+26$
Bin 5: 30

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Exercise E, Question 2

Question:

Laura wishes to record the following television programmes onto DVDs, each of which can hold up to 3 hours of programmes.

Programme	A	B	C	D	E	F	G	H	I	J	K	L	M
Length (minutes)	30	30	30	45	45	60	60	60	60	75	90	120	120

a Apply the first-fit algorithm, in the order A to M, to determine the number of DVDs that need to be used. State which programmes should be recorded on each disc.

b Repeat part **a** using the first-fit decreasing algorithm.

c Is your answer to part **b** optimal? Give a reason for your answer.

Laura finds that her DVDs will only hold up to 2 hours of programmes.

d Use the full-bin algorithm to determine the number of DVDs she needs to use. State which programmes should be recorded on each disc.

Solution:

a Bin 1: A(30) + B(30) + C(30) + D(45) + E(45)
Bin 2: F(60) + G(60) + H(60)
Bin 3: I(60) + J(75)
Bin 4: K(90)
Bin 5: L(120)
Bin 6: M(120)

b Bin 1: M(120) + I(60)
Bin 2: L(120) + H(60)
Bin 3: K(90) + J(75)
Bin 4: G(60) + F(60) + E(45)
Bin 5: D(45) + C(30) + B(30) + A(30)

c $$\text{Lower bound} = \frac{30+30+30+45+45+60+60+60+60+75+90+120+120}{180}$$
$$= \frac{825}{180}$$
$$= 4.5 \text{ so 5 tapes needed at least}$$

Since a minimum of 5 tapes are needed and **b** uses 5 tapes it is optimal

d For example

$$\left.\begin{array}{l}\text{Bin 1: M(120)}\\ \text{Bin 2: L(120)}\\ \text{Bin 3: A(30) + K(90)}\\ \text{Bin 4: F(60) + G(60)}\\ \text{Bin 5: H(60) + I(60)}\\ \text{Bin 6: J(75) + E(45)}\end{array}\right\}\text{ Full bins}$$
Bin 7: B(30) + C(30) + D(45)

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Exercise E, Question 3

Question:

A small ferry has three car lanes, each 30 m long. There are 10 vehicles waiting to use the ferry.

	Vehicle	Length (m)
A	car	4 m
B	car + trailer	7 m
C	lorry	13 m
D	van	6 m
E	lorry	13 m

	Vehicle	Length (m)
F	car	4 m
G	lorry	12 m
H	lorry	14 m
I	van	6 m
J	lorry	11 m

a Apply the first-fit algorithm, in the order A to J. Is it possible to load all the vehicles using this method?

b Apply the first-fit decreasing algorithm. Is it possible to load all the vehicles using this method?

c Use full-bin packing to load all of the vehicles.

Solution:

a Bin 1: $A(4) + B(7) + C(13) + D(6)$

Bin 2: $E(13) + F(4) + G(12)$

Bin 3: $H(14) + I(6)$

Bin 4: $J(11)$

So not possible is 3 bins.

b Re-ordering list

H(14) C(13) E(13) G(12) J(11) B(7) D(6) I(6) A(4) F(4)

Bin 1: $H(14) + C(13)$

Bin 2: $E(13) + G(12) + A(4)$

Bin 3: $J(11) + B(7) + D(6) + I(6)$

Bin 4: $F(4)$

Still not possible in 3 bins

c Bin 1: $H(14) + G(12) + A(4)$

Bin 2: $C(13) + E(13) + F(4)$

Bin 3: $J(11) + B(7) + D(6) + I(6)$

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Exercise E, Question 4

Question:

The ground floor of an office block is to be fully recarpeted, with specially made carpet which incorporate the firm's logo. The carpet comes in rolls of 15 m.

The following lengths are required.

A	3 m	D	4 m	G	5 m	J	7 m
B	3 m	E	4 m	H	5 m	K	8 m
C	4 m	F	4 m	I	5 m	L	8 m

Determine how the lengths should be cut from the rolls using

a the first-fit algorithm A to L,

b the first-fit decreasing algorithm,

c full-bin packing.

In each case, state the number of rolls used and the amount of wasted carpet.

Solution:

a Bin 1: A(3) B(3) C(4) D(4)
Bin 2: E(4) F(4) G(5)
Bin 3: H(5) I(5)
Bin 4: J(7) K(8)
Bin 5: L(8)
5 rolls used and 15 m wasted.

b For example
Bin 1: L(8) J(7)
Bin 2: K(8) I(5)
Bin 3: H(5) G(5) F(4)
Bin 4: E(4) D(4) C(4) B(3)
Bin 5: A(3)
5 rolls used and 15 m wasted.

c For example
Bin 1: L(8) J(7)
Bin 2: G(5) H(5) I(5)
Bin 3: C(4) D(4) E(4) B(3)
Bin 4: K(8) F(4) A(3)
4 rolls used and no wastage.

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Exercise E, Question 5

Question:

Eight computer programs need to be recorded onto 40 MB discs.
The size of each program is given below.

Programme	A	B	C	D	E	F	G	H
Size (MB)	8	16	17	21	22	24	25	25

a Use the first-fit decreasing algorithm to determine which programs should be recorded onto each disc.

b Calculate a lower bound for the number of discs needed.

c Explain why it is not possible to record these programs on the number of discs found in part **b**.

Consider the programs over 20 MB in size.

Solution:

a Bin 1: H(25) A(8)
Bin 2: G(25)
Bin 3: F(24) B(16)
Bin 4: E(22) C(17)
Bin 5: D(21)

b $\frac{8+16+17+21+22+24+25+25}{40}=\frac{158}{40}=3.95$

$\therefore$ Lower bound is 4

c There are 5 programs over 20 MB. It is not possible for any two of these to share a bin. So at least 5 bins will be needed, so 4 will be insufficient.

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Exercise F, Question 1

Question:

Use the bubble-sort algorithm to sort, in ascending order, the list:

27 15 2 38 16 1

giving the state of the list at each stage. [*E*]

Solution:

Bubbling left to right

Initial list	27	15	2	38	16	1
1st pass	15	2	27	16	1	38
2nd pass	2	15	16	1	27	38
3rd pass	2	15	1	16	27	38
4th pass	2	1	15	16	27	38
5th pass	1	2	15	16	27	38

No further changes $\therefore$ sorted

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Exercise F, Question 2

Question:

a Use the bubble-sort algorithm to sort, in descending order, the list:
25 42 31 22 26 41
giving the state of the list on each occasion when two values are interchanged.

b Find the *maximum* number of interchanges needed to sort a list of six pieces of data using the bubble-sort algorithm. [*E*]

Solution:

a Bubbling left to right

Initial list	25	42	31	22	26	41
1st pass	42	31	25	26	41	22
2nd pass	42	31	26	41	25	22
3rd pass	42	31	41	26	25	22
4th pass	42	41	31	26	25	22

No further changes $\therefore$ sorted

b 15

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Exercise F, Question 3

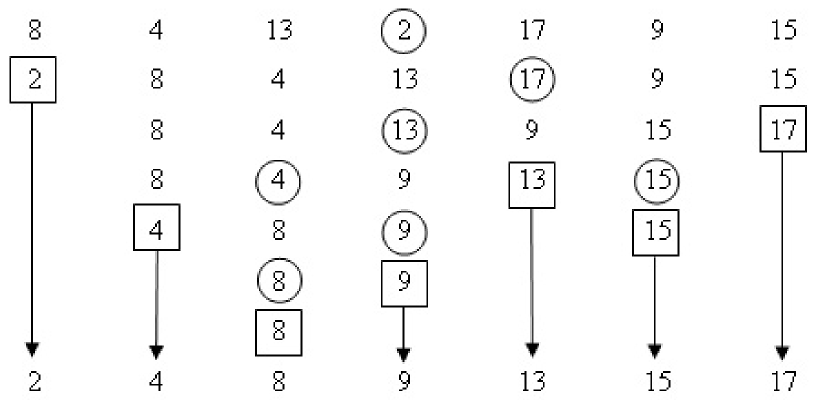
Question:

8 4 13 2 17 9 15

This list of numbers is to be sorted into desending order.

Perform a quick sort to obtain the sorted list, giving the state of the list after each rearrangement. [*E*]

Solution:



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Exercise F, Question 4

Question:

111 103 77 81 98 68 82 115 93

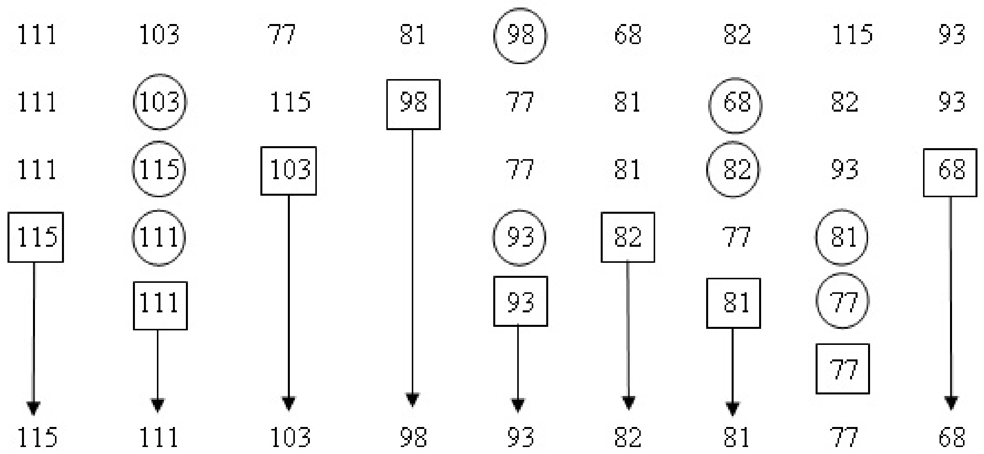
a The list of numbers above is to be sorted into descending order. Perform a quick-sort to obtain the sorted list, giving the state of the list after each rearrangement and indicating the pivot elements used.

b **i** Use the first-fit decreasing bin-packing algorithm to fit the data into bins of size 200.

ii Explain how you decided in which bin to place the number 77. [*E*]

Solution:

a



b **i** Bin1: $115 + 82$
Bin2: $111 + 81$
Bin3: $103 + 93$
Bin4: $98 + 77$
Bin5: 68

ii No room in bin 1(3 left) or bin 2 (8 left) or bin 3 (4 left) but room in bin 4.

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Exercise F, Question 5

Question:

Trishna wishes to video eight television programmes. The lengths of the programmes, in minutes, are:

75 100 52 92 30 84 42 60

Trishna decides to use 2-hour (120-minute) video tapes only to record all of these programmes.

a Explain how to use a first-fit decreasing bin-packing algorithm to find the solution that uses the fewest tapes and determine the total amount of unused tape.

b Determine whether it is possible for Trishna to record an additional two 25-minute programmes on these 2-hour tapes, without using another video tape. [*E*]

Solution:

a Rank the times in descending order and use them in this order. Number the bins, starting at 1.
Place each recording time into the first available bin, starting with bin 1 each time.

100 92 84 75 60 52 42 30

Bin 1: 100
Bin 2: 92
Bin 3: 84 + 30
Bin 4: 75 + 42
Bin 5: 60 + 52

$$\begin{aligned}\text{unused tape} &= 5\times120-(100+92+84+75+60+52+42+30)\\ &= 600-535\\ &= 65\ \text{minutes}\end{aligned}$$

b There is room on tape 2 for one of the 25-minute programmes but no room on any tape for the second programme.

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Exercise F, Question 6

Question:

A DIY enthusiast requires the following 14 pieces of wood as shown in the table.

Length in metres	0.4	0.6	1	1.2	1.4	1.6
Number of pieces	3	4	3	2	1	1

The DIY store sells wood in 2 m and 2.4 m lengths. He considers buying six 2 m lengths of wood.

a Explain why he will not be able to cut all of the lengths he requires from these six 2 m lengths.

b He eventually decides to buy 2.4 m lengths. Use a first-fit decreasing bin-packing algorithm to show how he could use six 2.4 m lengths to obtain the pieces he requires.

c Obtain a solution that only requires five 2.4 m lengths. [*E*]

Solution:

a For example, the length total 12 m so no wastage is permitted. We are therefore seeking a full bin solution.

The two 1.2 m length can not be 'made up' to 2 m bins since these are only 3×0.4 m length. Two of these can be used to make a full bin, $1.2+0.4+0.4$, but the second 1.2 m can not be made up to 2 m since there is only 1 remaining 0.4 m length.

b Bin 1: $1.6+0.6$
Bin 2: $1.4+1$
Bin 3: $1.2+1.2$
Bin 4: $1+1+0.4$
Bin 5: $0.6+0.6+0.6+0.4$
Bin 6: 0.4

c For example
Bin 1: $1.6+0.4+0.4$
Bin 2: $1.4+1$
Bin 3: $1.2+1.2$
Bin 4: $1+1+0.4$
Bin 5: $0.6+0.6+0.6+0.6$

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Exercise F, Question 7

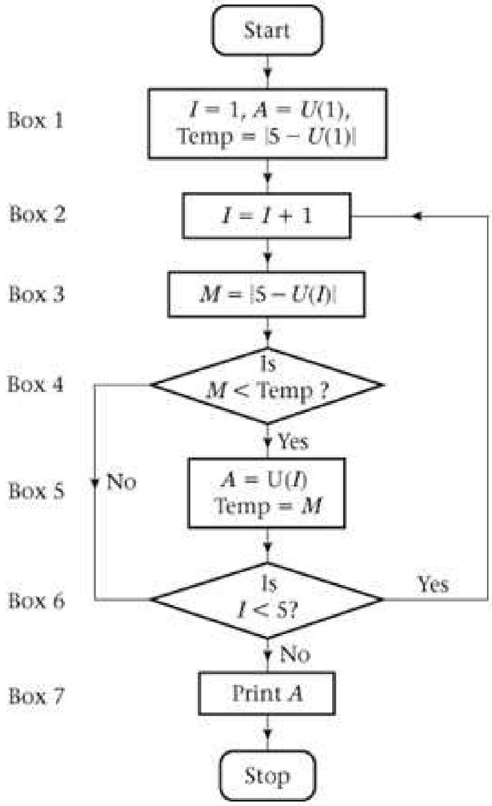
Question:

Note: This question uses the modulus function. If $x \neq y, |x-y|$ is the positive difference between x and y, e.g. $|5-6.1|=1.1$. The algorithm described by the flow chart below is to be applied to the five pieces of data below.

$U(1)=6.1, U(2)=6.9, U(3)=5.7, U(4)=4.8, U(5)=5.3$

a Obtain the final output of the algorithm using the five values given for $U(1)$ to $U(5)$.

b In general, for any set of values $U(1)$ to $U(5)$, explain what the algorithm achieves.



c If Box 4 in the flow chart is altered to
'Is M > Temp ?'
state what the algorithm now achieves. [*E*]

Solution:

a

	I	M	Box 4	A	Temp	Box 6
Initial conditions	1	-		6.1	1.1	
1st pass	2	1.9	No	6.1	1.1	Yes
2nd pass	3	0.7	Yes	5.7	0.7	Yes
3rd pass	4	0.2	Yes	4.8	0.2	Yes
4th pass	5	0.3	No			No

output 4.8

b It selects the number nearest to 5.

c It would select the number furthest from 5.

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Exercise A, Question 1

Question:

Draw a connected graph with

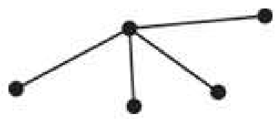
a one vertex of degree 4 and 4 vertices of degree 1,

b three vertices of degree 2, one of degree 3 and one of degree 1,

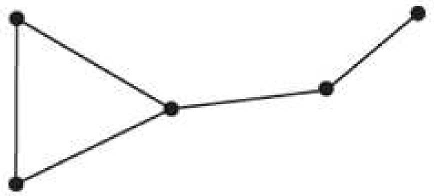
c two vertices of degree 2, two of degree 3 and one of degree 4.

Solution:

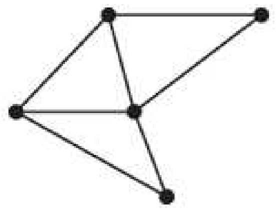
a For example



b



c



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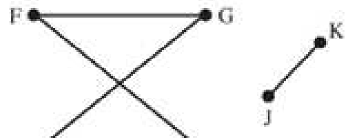
Exercise A, Question 2

Question:

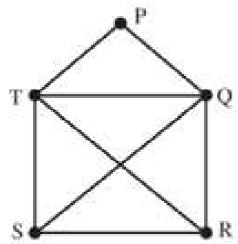
Which of the graphs below are not simple?

a

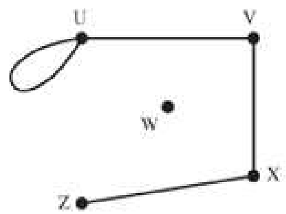
b



c



d



Solution:

a is not simple. There are two edges connecting C with D.
b and **c** are simple.
d is not simple. There is a loop attached to U.

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Exercise A, Question 3

Question:

In question 2, which graphs are not connected?

Solution:

a and **c** are connected.
b is not connected, there is no path from J to G, for example.
d is not connected, there is no path from W to Z, for example.

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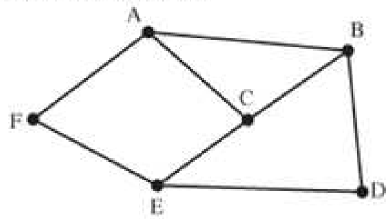
Exercise A, Question 4

Question:

From the graph, state



There are many correct answers to these questions.

a four paths from F to D,
b a cycle passing through F and D,
c the degree of each vertex.
Use the graph to
d draw a subgraph,
e confirm the handshaking lemma (that the sum of the degrees is equal to twice the number of edges).

A lemma is a mathematical fact used as a stepping stone to more important results.

Solution:

a Any four of these

F A B D F E D
F A C B D F E C B D
F A B C E D F E C A B D
F A C E D

b Here are examples. (These all start at F, but you could start at any point.)
There are others.

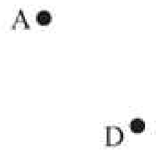
F A B D E F F A B D B A F F A C E D E C A F
F E D B A F F E D E F F A C E D E F
F A C B D E F F A C B D B C A F F E D E C A F
F E D B C A F F E C B D B C E F F E C B D E F
F E D B C E F

c

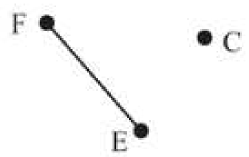
Vertex	A	B	C	D	E	F
Degree	3	3	3	2	3	2

d Here are examples. (There are many others.)

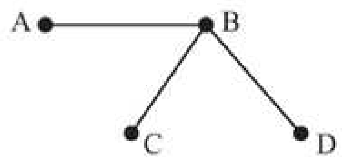
i



ii



iii



e Sum of degrees $= 3+3+3+2+3+2 = 16$
number of edges $= 8$
sum of degrees $= 2 \times$ number of edges for this graph

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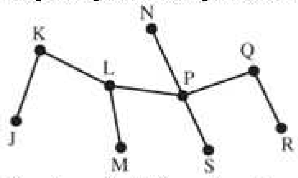
Exercise A, Question 5

Question:

a Repeat question 4 parts **c**, **d** and **e** using this graph.



b Confirm that there is only one path between any two vertices.

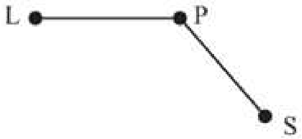
Solution:

a

Vertex	J	K	L	M	N	P	Q	R	S
Degree	1	2	3	1	1	4	2	1	1

Here are some possible subgraphs (there are many others).

i



ii

iii

Sum of degrees $= 1+2+3+1+1+4+2+1+1 = 16$

number of edges $= 8$

sum of degrees $= 2 \times$ number of edges for this graph

b This graph is a tree so there will only be one path between any two vertices.

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Exercise A, Question 6

Question:

Show that it is possible to draw a graph with

a an even number of vertices of even degree,

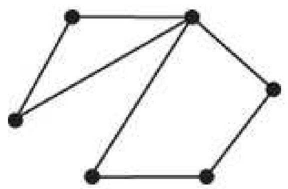
b an odd number of vertices of even degree.

It is not possible to draw a graph with an odd number of vertices of odd degree. Explain why not.

Use the handshaking lemma.

Solution:

a For example



6 vertices all even
5 of degree 2
1 of degree 4

b For example

3 vertices all even, all of order 2

The sum of degrees $= 2 \times$ number of edges, so the sum of degrees must be even. Any vertices of odd degree must therefore 'pair up'. So there must be an even number of vertices of odd degree.

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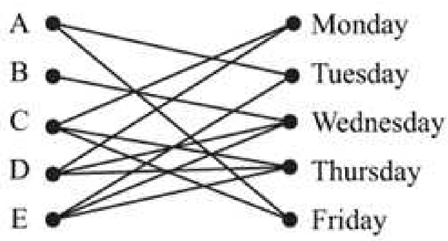
Exercise A, Question 7

Question:

Five volunteers, Ann, Brian, Conor, Dave and Eun Jung are going to run a help desk from Monday to Friday next week.
One person is required each day.
Ann is available on Tuesday and Friday,
Brian is available on Wednesday,
Conor is available on Monday, Thursday and Friday,
Dave is available on Monday, Wednesday and Thursday,
Eun Jung is available on Tuesday, Wednesday and Thursday.
Draw a graph to model this situation.

Look at Example 3.

Solution:



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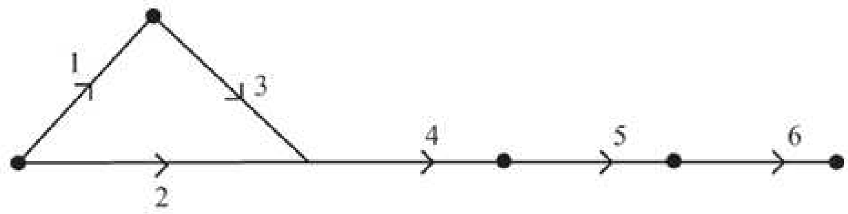
Exercise A, Question 8

Question:

A project consists of six activities 1, 2, 3, 4, 5 and 6.
1 and 2 can start immediately, but 3 cannot start until 1 is completed.
4 cannot start until both 2 and 3 are complete, 5 cannot start until 4 is complete and 6 cannot start until 5 is complete.
Draw a digraph to model this situation.

Look at Example 5.

Solution:



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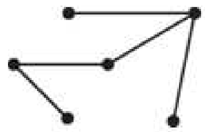
Exercise B, Question 1

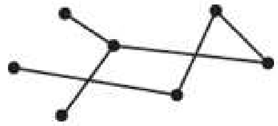
Question:

State which of the following graphs are trees.

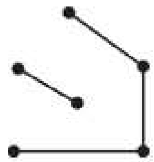
a



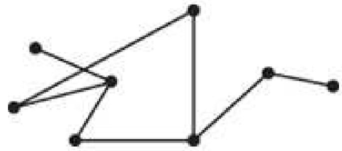
b



c



d



Solution:

a and **b** are trees.
c is not a tree, it is not a connected graph.
d is not a tree, it contains a cycle.

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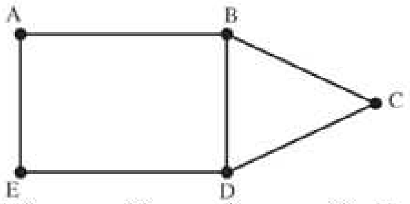
Exercise B, Question 2

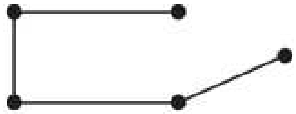
Question:



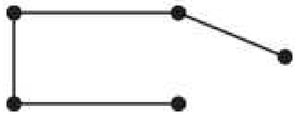
There are 11 spanning trees for the graph above. See how many you can find.

Solution:

i

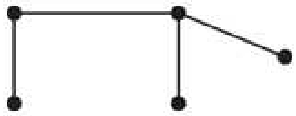


ii



iii

iv



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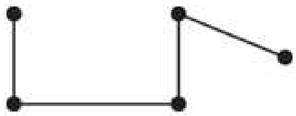
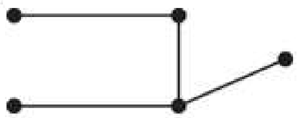
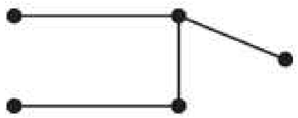


vi

vii



viii



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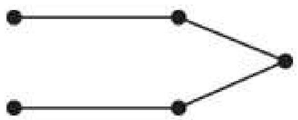
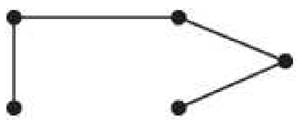


x



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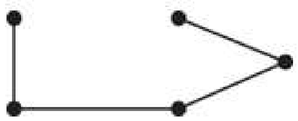

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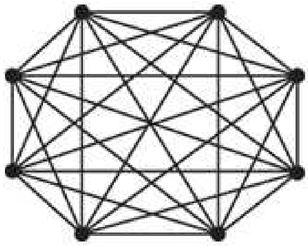
Exercise B, Question 3

Question:

Draw k_8.
What is the degree of each vertex in the graph k_n?

Solution:



k_8

Each vertex will have a degree of $(n-1)$.

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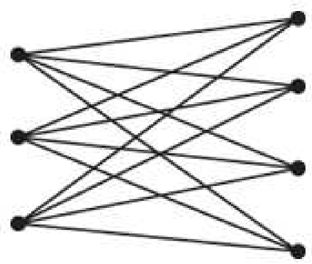
Exercise B, Question 4

Question:

Draw $k_{3,4}$.

How many edges would be in $k_{n,m}$?

Solution:



$k_{3,4}$

There will be nm edges.

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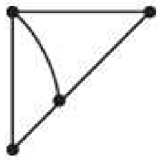
Exercise B, Question 5

Question:

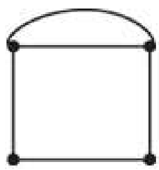
Which of these graphs are isomorphic?

a



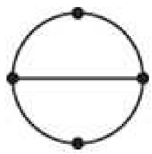
b

c

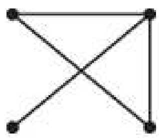


d

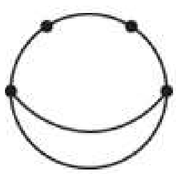
e



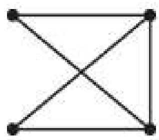
f



g



h



i

Solution:

a, **e** and **h** are isomorphic.
b and **i** are isomorphic.
c and **g** are isomorphic.
d and **f** are isomorphic.

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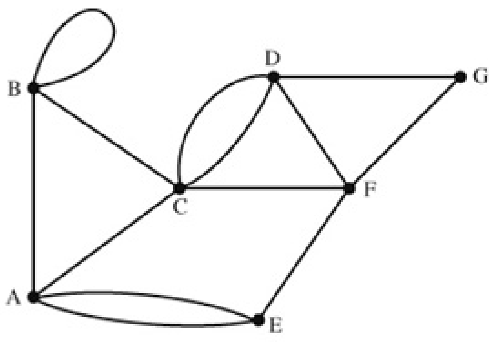
Exercise B, Question 6

Question:



Use an adjacency matrix to represent the graph above.

Solution:

	A	B	C	D	E	F	G
A	0	1	1	0	2	0	0
B	1	2	1	0	0	0	0
C	1	1	0	2	0	1	0
D	0	0	2	0	0	1	1
E	2	0	0	0	0	1	0
F	0	0	1	1	1	0	1
G	0	0	0	1	0	1	0

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Exercise B, Question 7

Question:

Draw a graph corresponding to each adjacency matrix.

a

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	1	1
C	0	1	0	2	0
D	1	1	2	0	1
E	0	1	0	1	0

b

	A	B	C	D
A	0	1	0	1
B	1	0	0	1
C	0	0	2	1
D	0	0	1	1

c

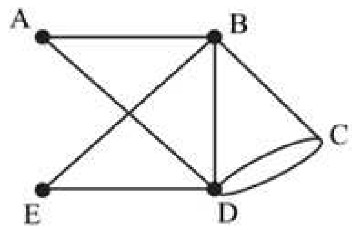
	A	B	C	D
A	0	1	0	1
B	1	0	1	1
C	0	1	0	1
D	1	1	1	0

d

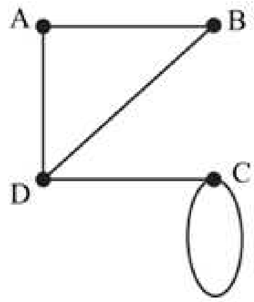
	A	B	C	D
A	0	2	0	1
B	2	0	1	0
C	0	1	0	1
D	1	0	1	0

Solution:

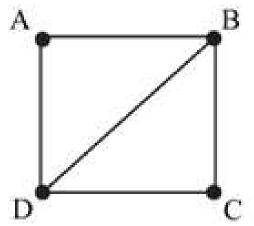
a



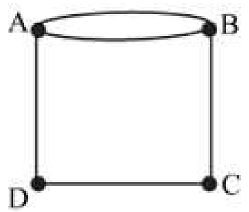
b



c



d



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Exercise B, Question 8

Question:

Which graphs in Question **5** could be described by the adjacency matrices in Question 7**c** and **d**?

Solution:

7**c** could describe 5**a**, **e** and **h**.
7**d** could describe 5**c** and **g**.

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Exercise B, Question 9

Question:

Draw the network corresponding to each distance matrix.

a

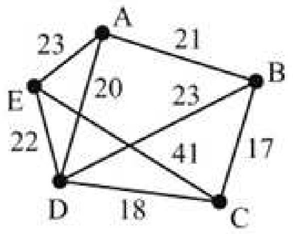
	A	B	C	D	E
A	–	21	–	20	23
B	21	–	17	23	–
C	–	17	–	18	41
D	20	23	18	–	22
E	23	–	41	22	–

b

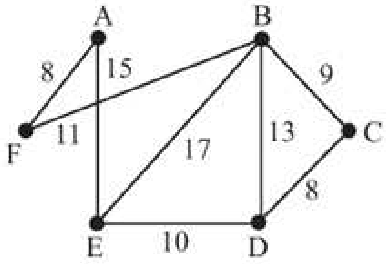
	A	B	C	D	E	F
A	–	–	–	–	15	8
B	–	–	9	13	17	11
C	–	9	–	8	–	–
D	–	13	8	–	10	–
E	15	17	–	10	–	–
F	8	11	–	–	–	–

Solution:

a



b



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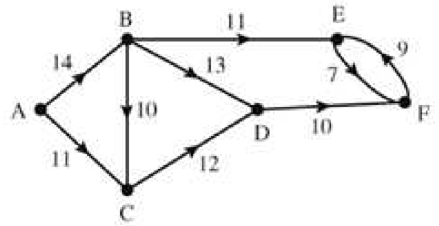
Exercise B, Question 10

Question:



Use a distance matrix to represent the directed network above.

Solution:

	A	B	C	D	E	F
A	–	14	11	–	–	–
B	–	–	10	13	11	–
C	–	–	–	12	–	–
D	–	–	–	–	–	10
E	–	–	–	–	–	7
F	–	–	–	–	9	–

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Exercise C, Question 1

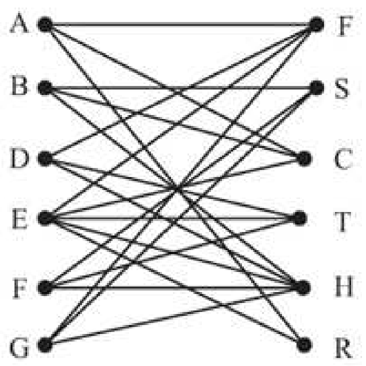
Question:

A group of 6 children, Ahmed, Bronwen, Di, Eddie, Fiona and Gary, were asked which of 7 sports, football, swimming, cricket, tennis, hockey and rugby, they enjoyed playing. The results are shown in the table below.

Ahmed	football	cricket	rugby		
Bronwen	swimming	cricket	hockey		
Di	football	tennis	hockey		
Eddie	football	cricket	tennis	hockey	rugby
Fiona	tennis	swimming	hockey		
Gary	football	swimming	hockey		

Draw a bipartite graph to show this information.

Solution:



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Exercise C, Question 2

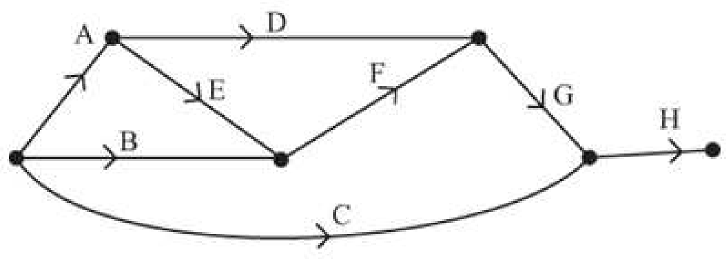
Question:

A project involves eight activities A, B, C, D, E, F, G and H, some of which cannot be started until others are completed. The table shows the tasks that need to be completed before the activity can start. For example, activity F cannot start until both B and E are completed.

Activity	Activity that must be completed
A	–
B	–
C	–
D	A
E	A
F	B E
G	D F
H	C G

Draw a digraph to represent this information.

Solution:



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Exercise C, Question 3

Question:

a Draw a graph with eight vertices, all of degree 1.

b Draw a graph with eight vertices, all of degree 2, so that the graph is

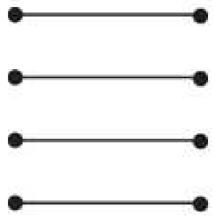
i connected and simple

ii not connected and simple

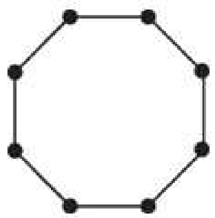
iii not connected and not simple.

Solution:

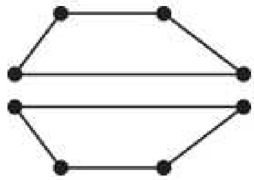
a



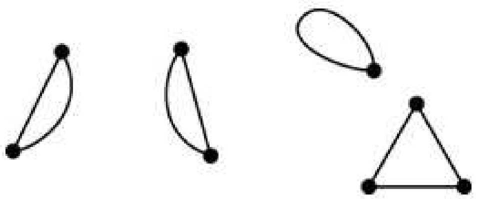
b **i**



ii



iii



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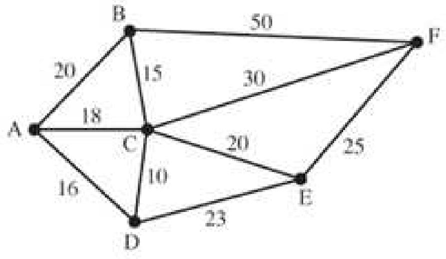
Exercise C, Question 4

Question:

Use a distance matrix to represent the network below.



Solution:

	A	B	C	D	E	F
A	–	20	18	16	–	–
B	20	–	15	–	–	50
C	18	15	–	10	20	30
D	16	–	10	–	23	–
E	–	–	20	23	–	25
F	–	50	30	–	25	–

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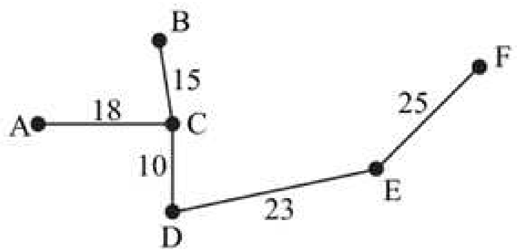
Exercise C, Question 5

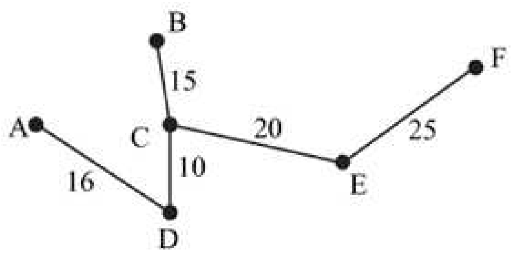
Question:

Find two spanning trees for the graph in Question **4**.

Solution:

Here are examples. (There are many other solutions.)





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Exercise C, Question 6

Question:

Write down a formula connecting the number of edges, E, in a spanning tree with V vertices.

Solution:

$E = V - 1$ (or $V = E + 1$)

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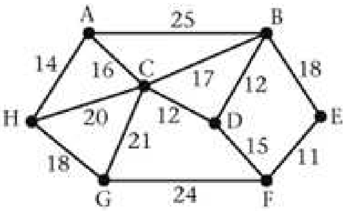
Exercise A, Question 1

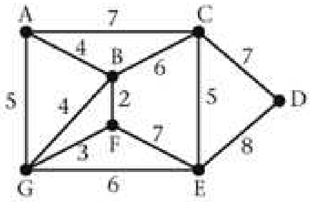
Question:

Use Kruskal's algorithm to find minimum spanning trees for each of these networks. State the weight of each tree. You must list the arcs in the order in which you consider them.

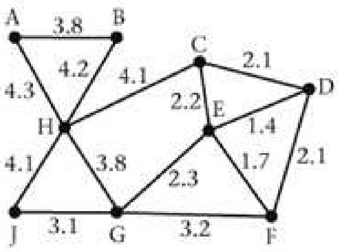
a



b

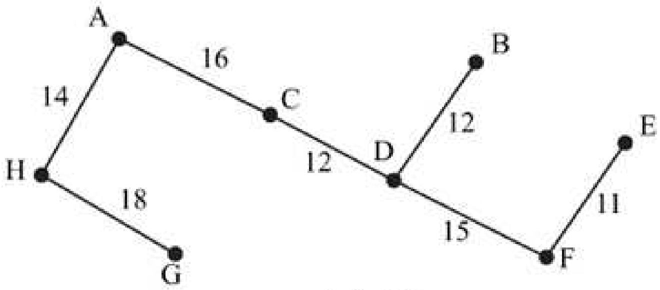


c



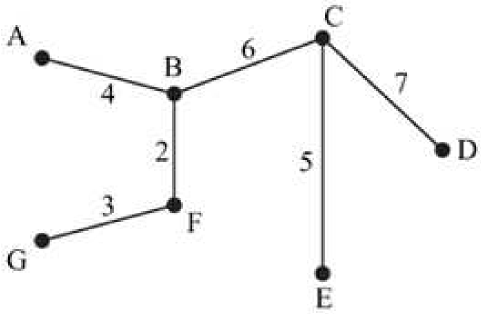
Solution:

a EF (11) add to tree
BD (12) add to tree
CD (12) add to tree
AH (14) add to tree
DF (15) add to tree
AC (16) add to tree
BC (17) reject
GH (18) add to tree
BE (18)
CH (20)
CG (21) reject all remaining arcs.
FG (24)
AB (25)



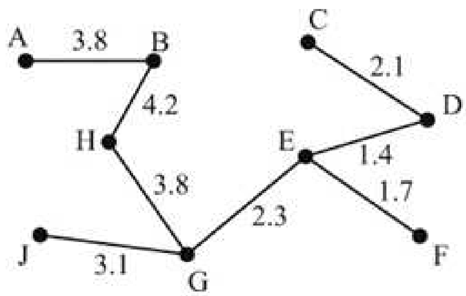
weight: 98

b BF (2) add to tree
FG (3) add to tree
AB (4) add to tree
BG (4) reject
AG (5) reject
CE (5) add to tree
BC (6) add to tree
EG (6) reject
AC (7) reject
CD (7) add to tree
EF (7) reject all remaining arcs
DE (8)



weight: 27

c DE (1.4) add to tree
EF (1.7) add to tree
CD (2.1) add to tree
DF (2.1) reject
CE (2.2) reject
EG (2.3) add to tree
GJ (3.1) add to tree
FG (3.2) reject
AB (3.8) add to tree
GH (3.8) add to tree
GH (4.1) reject
HJ (4.1) reject
BH (4.2) add to tree
AH (4.3) reject



weight: 22.4

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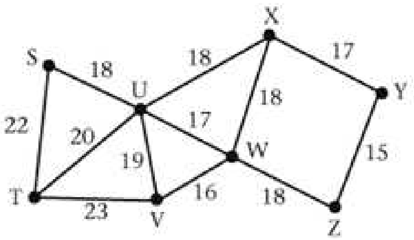
Exercise A, Question 2

Question:

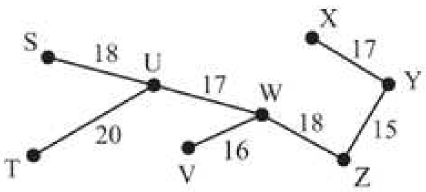
Use Kruskal's algorithm to find the three possible minimum connectors (MSTs) for this network. You must list the arcs in the order in which you consider them.



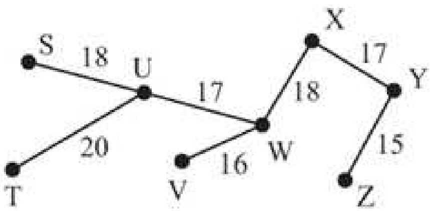
Solution:

YZ (15) add to tree
VW (16) add to tree
XY (17) add to tree
UW (17) add to tree

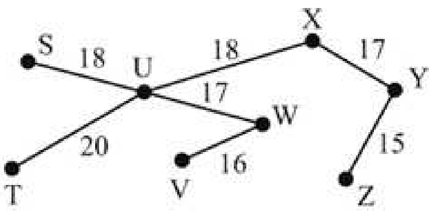
WZ (18), WX (18), UX (18) — reject 2 of these and add one to tree
SU (18) add to tree
UV (19) reject
TU (20) add to tree
ST (22) reject
TV (23) reject



solution 1



solution 2



solution 3

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Exercise A, Question 3

Question:

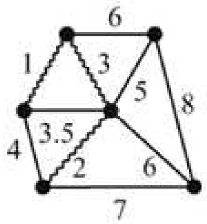
Draw a network in which

a the three shortest arcs from part of the minimum connector (MST),

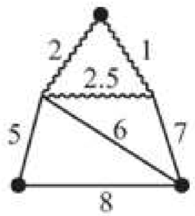
b not all of the three shortest arcs from part of the minimum connector.

Solution:

a For example



b For example



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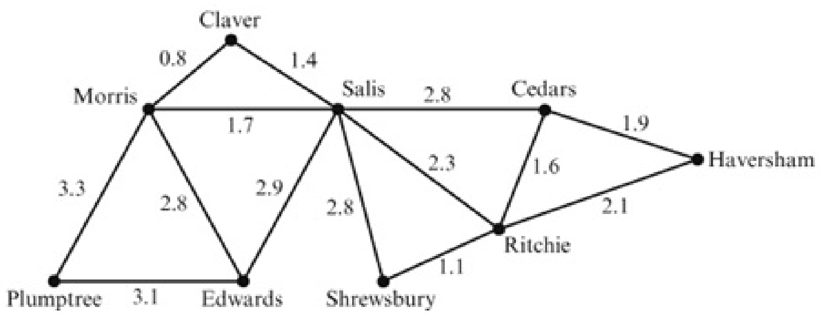
Exercise A, Question 4

Question:

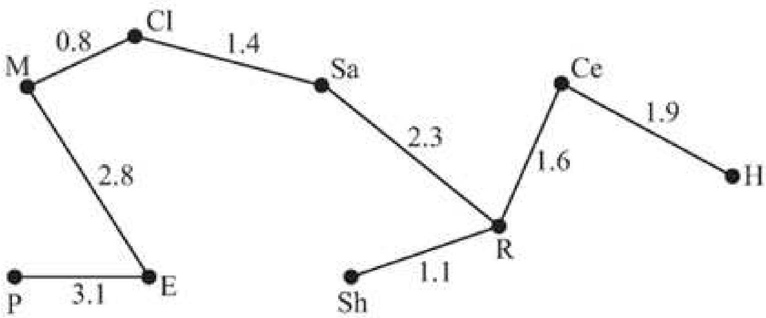
The diagram shows nine estates and the distances, in km, between them. A cable TV company wishes to link up the estates. Find a minimum spanning tree for the network using Kruskal's algorithm. You must list the arcs in the order in which you consider them. State the weight of your tree.

a



Solution:

Cl − M (0.8) add to tree
R − Sh (1.1) add to tree
Cl − Sa (1.4) add to tree
Ce − R (1.6) add to tree
Sa − M (1.7) reject
Ce − H (1.9) add to tree
H − R (2.1) reject
R − Sa (2.3) add to tree
Ce − Sa (2.8) reject
Sa − Sh (2.8) reject
E − M (2.8) add to tree
E − Sa (2.9) reject
E − P (3.1) add to tree
M − P (3.3) reject



weight: 15

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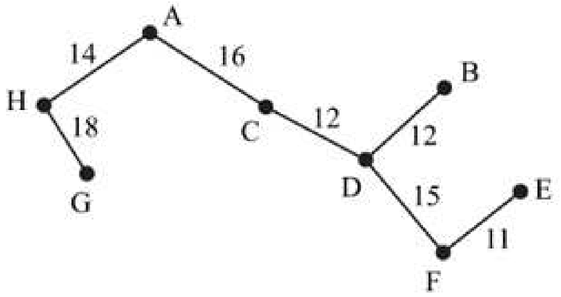
Exercise B, Question 1

Question:

Repeat Question **1** in Exercise 3A using Prim's algorithm. Start at vertex A each time.

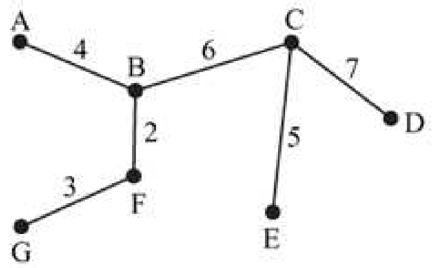
Solution:

a Arcs must be chosen in this order:
AH, AC, CD, BD, DF, FE, GH



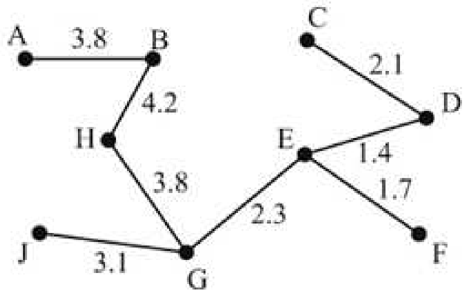
weight: 98

b Arcs must be chosen in this order:
BF, FG, AB, BC, CE, CD



weight: 27

c Arcs must be chosen in this order:
AB, BH, GH, GE, DE, EF, CD, GJ



weight: 22.4

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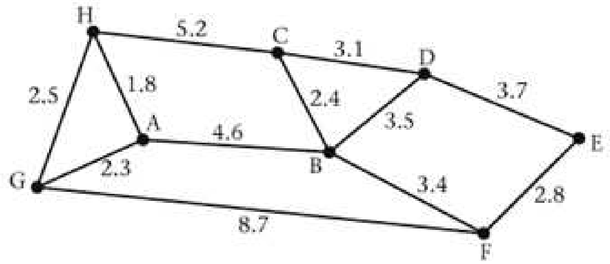
Exercise B, Question 2

Question:



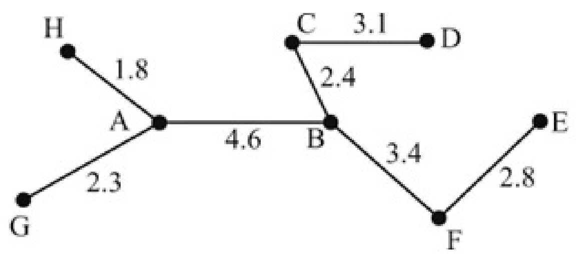
The network shows the distance, in kilometres, between eight weather monitoring stations. The eight stations need to be linked together with underground cables.

a Use Prim's algorithm, starting at A, to find a minimum spanning tree. You must make your order of arc selection clear.

b Given that cable costs £850 per kilometre to lay, find the cost of linking these weather stations.

Solution:

a Arcs must be chosen in this order
AH, AG, AB, BC, CD, BF, FE



b $\text{cost} = 20.4 \times 850$

$= £17\,340$

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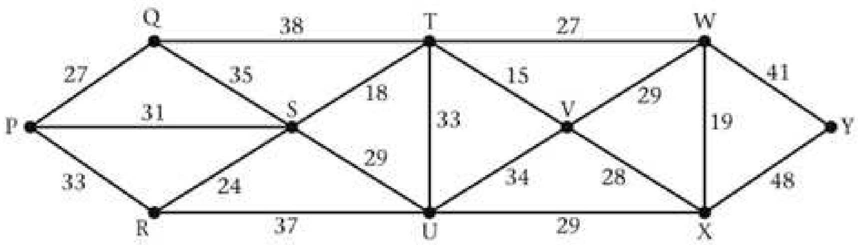
Exercise B, Question 3

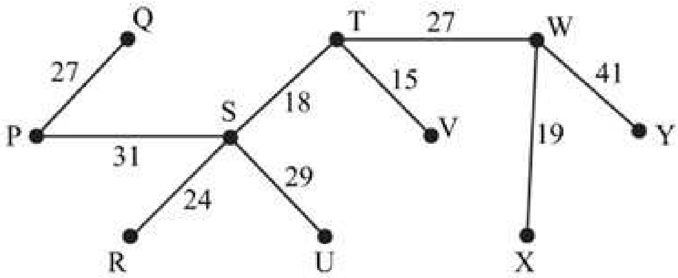
Question:



The network shows ten villages and the costs, in thousands of pounds, of connecting them with a new energy supply.
Use Prim's algorithm starting at P, to find the minimum cost energy supply network that would connect all ten villages.
Draw your minimum connector and state its cost.

Solution:

Arcs must be chosen in this order
either
PS, ST, TV, RS, TW, WX, PQ, SU, WY
or
PS, ST, TV, RS, PQ, TW, WX, SU, WY.
Cost: £231 000



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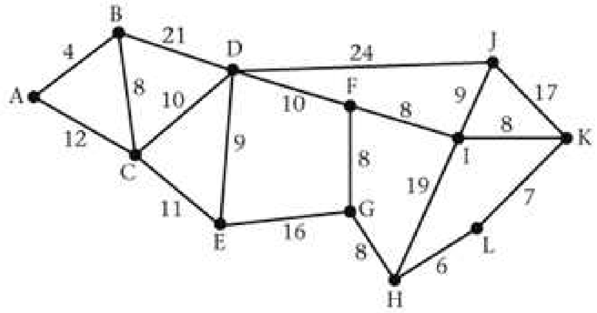
Exercise B, Question 4

Question:

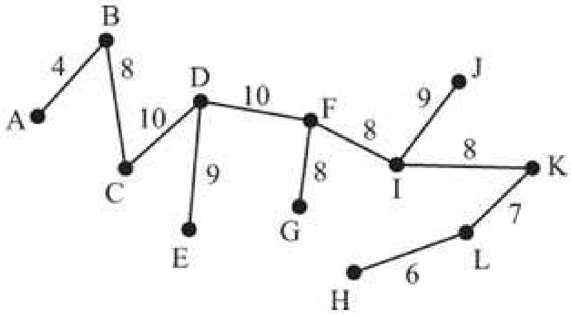
Use Prim's algorithm, starting at A, to find four distance minimum connectors for the network below. In each case draw your spanning tree, and make your order of arc selection clear.



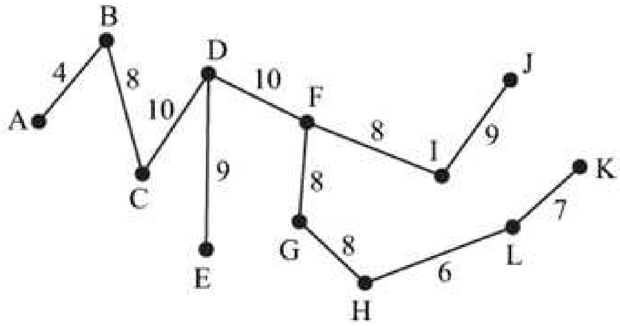
Solution:

Solution 1

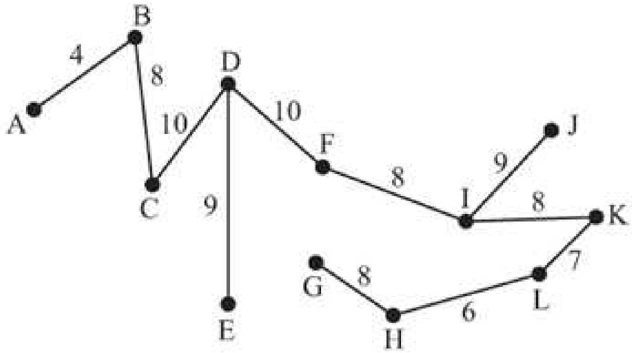
AB, BC, CD, DE, DF
then *either*
FG FI IK KL HL IJ
or FI FG IK KL HL IJ
or FI IK KL HL FG IJ



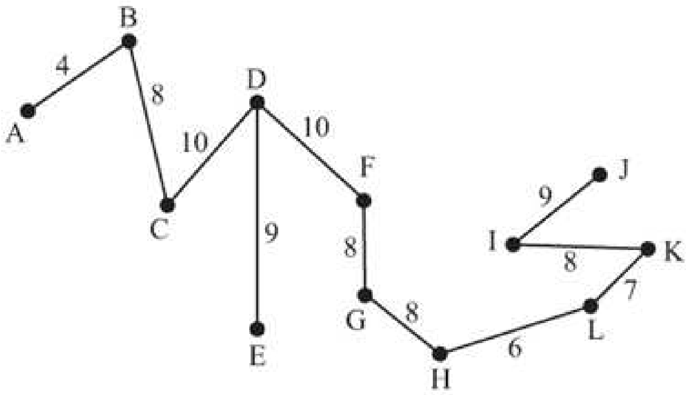
Solution 2
AB BC CD DE DF
then *either* FI FG GH HL LK IJ
or FG GH HL LK FI IJ
or FG FI GH HL LK IJ



Solution 3
AB BC CD DE DF FI
IK KL HL GH IJ



Solution 4
AB BC CD DE DF FG
GH HL KL IK IJ



Solutionbank D1
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Exercise C, Question 1

Question:

Apply Prim's algorithm to the distance matrices below. List the arcs in order of selection and state the weight of your tree.

a

	A	B	C	D	E	F
A	–	15	20	34	25	9
B	15	–	36	38	28	14
C	20	36	–	43	38	22
D	34	38	43	–	26	40
E	25	28	38	26	–	31
F	9	14	22	40	31	–

b

	R	S	T	U	V
R	–	28	30	31	41
S	28	–	16	19	43
T	30	16	–	22	41
U	31	19	22	–	37
V	41	43	41	37	–

Solution:

a

	↓1	↓3	↓4	↓6	↓5	↓2
	A	B	C	D	E	F
A	–	15	20	34	25	9
B	15	–	36	38	28	14
C	20	36	–	43	38	22
D	34	38	43	–	26	40
E	25	28	38	26	–	31
F	9	14	22	40	31	–

Arcs in order
AF (9)
FB (14)
AC (20)
AE (25)
DE (26)

$$\text{weight} = 9 + 14 + 20 + 25 + 26$$
$$= 94$$

b

	↓1	↓2	↓3	↓4	↓5
	R	S	T	U	V
R	–	28	30	31	41
S	28	–	16	19	43
T	30	16	–	22	41
U	31	19	22	–	37
V	41	43	41	37	–

Arcs in order
RS (28)
ST (16)
SU (19)
UV (37)

$$\text{weight} = 28 + 16 + 19 + 37$$
$$= 100$$

Solutionbank D1
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Exercise C, Question 2

Question:

	Birmingham	Nottingham	Lincoln	Stoke	Manchester
Birmingham	–	164	100	49	88
Nottingham	164	–	37	56	74
Lincoln	100	37	–	90	86
Stoke	49	56	90	–	44
Manchester	88	74	86	44	–

The table shows the distance, in miles, between 5 cities. It is intended to link these 5 cities to a transit system.
Use Prim's algorithm, starting at Birmingham, to find a minimum spanning tree for this network. You must list the arcs in order of selection and state the weight of your tree.

Solution:

	↓1 B	↓4 N	↓5 L	↓2 S	↓3 M
B	–	164	100	49	88
N	164	–	37	56	74
L	100	37	–	90	86
S	49	56	90	–	44
M	88	74	86	44	–

Arcs in order
BS (49)
SM (44)
SN (56)
NL (37)
weight = 186

Solutionbank D1

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Exercise C, Question 3

Question:

	A	B	C	D	E	F	G	H
A	–	84	53	35	–	47	–	42
B	84	–	71	113	142	61	75	–
C	53	71	–	–	–	–	59	–
D	35	113	–	–	58	67	151	–
E	–	142	–	58	–	168	159	48
F	47	61	–	67	168	–	–	73
G	–	75	59	151	159	–	–	52
H	42	–	–	–	48	73	52	–

The table shows the costs, in euros per 1000 words, of translating DVD player instruction manuals between eight languages.

a Use Prim's algorithm, starting from D, to find the cost of translating an instruction manual of 3000 words from D into the seven other languages.

b Draw your minimum spanning tree.

From the table we see that it costs 159 euros per 1000 words to translate from language E to G. A manual is written in language E and needs to be translated into language G.

c Give a reason why
 i it might be decided *not* to translate directly from E to G,
 ii it might be decided to translate directly.

Solution:

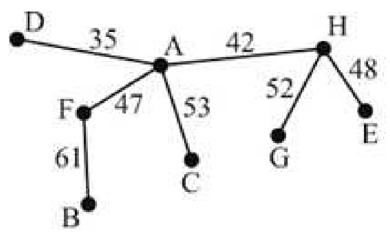
a

	↓2	↓8	↓7	↓1	↓5	↓4	↓6	↓3
	A	B	C	D	E	F	G	H
A	–	84	53	35	–	47	–	42
B	84	–	71	113	142	61	75	–
C	53	71	–	–	–	–	59	–
D	35	113	–	–	58	67	151	–
E	–	142	–	58	–	168	159	48
F	47	61	–	67	168	–	–	73
G	–	75	59	151	159	–	–	52
H	42	–	–	–	48	73	52	–

Arcs in order
DA (35)
AH (42)
AF (47)
HE (48)
HG (52)
AC (53)
FB (61)

weight $= 338$
$\therefore$ cost $= 3 \times 338$
$= €\,1014$

b



c **i** It is cheaper to translate from E to H then from H to G at a cost of $48 + 52 = 100$ euro rather than 159 euro per 1000 words.

ii A direct translation is likely to be more accurate than a translation via another language.

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Exercise C, Question 4

Question:

	X	A	B	C	D	E	F	G	H	I
X	–	65	80	89	74	26	71	41	41	74
A	65	–	27	41	22	37	20	29	25	43
B	80	27	–	30	24	55	16	46	40	42
C	89	41	30	–	50	84	24	70	49	26
D	74	22	24	50	–	51	35	34	47	63
E	26	37	55	84	51	–	52	18	23	68
F	71	20	16	24	35	52	–	45	31	27
G	41	29	46	70	34	18	45	–	25	64
H	41	25	40	49	47	23	31	25	–	44
I	74	43	42	26	63	68	27	64	44	–

The table shows the distances, in miles, between nine oil rigs and the depot X. Pipes are to be laid to connect the rigs and the depot.

a Use Prim's algorithm, starting at X, to find a minimum connector for the network. You must make the order of arc selection clear.

Oil rig A exhausts its supply and is closed down.

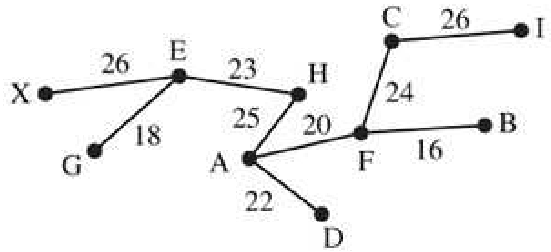
b Use Prim's algorithm to find a minimum connector excluding A. You must make the order of arc selection clear.

Solution:

a

	1	5	7	9	8	2	6	3	4	10
	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
	X	A	B	C	D	E	F	G	H	I
X	–	65	80	89	74	26	71	41	41	74
A	65	–	27	41	22	37	20	29	25	43
B	80	27	–	30	24	55	16	46	40	42
C	89	41	30	–	50	84	24	70	49	26
D	74	22	24	50	–	51	35	34	47	63
E	26	37	55	84	51	–	52	18	23	68
F	71	20	16	24	35	52	–	45	31	27
G	41	29	46	70	34	18	45	–	25	64
H	41	25	40	49	47	23	31	25	–	44
I	74	43	42	26	63	68	27	64	44	–

order of arcs
XE (26)
EG (18)
EH (23)
HA (25)
AF (20)
FB (16)
AD (22)
FC (24)
CI (26)

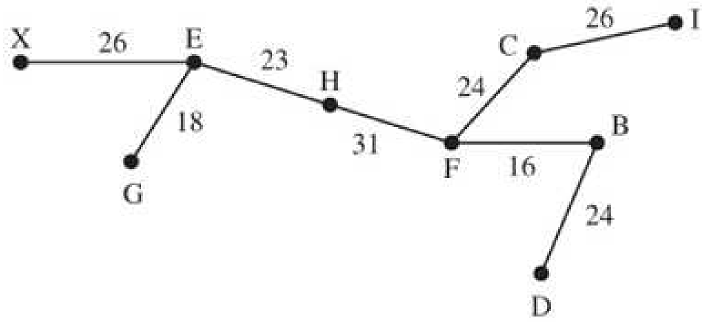


b

	1	6	7 or 8	8 or 9	2	5	3	4	9
	↓	↓	↓	↓	↓	↓	↓	↓	↓
	X	B	C	D	E	F	G	H	I
X	–	80	89	74	26	71	41	41	74
B	80	–	30	24	55	16	46	40	42
C	89	30	–	50	84	24	70	49	26
D	74	24	50	–	51	35	34	47	63
E	26	55	84	51	–	52	18	23	68
F	71	16	24	35	52	–	45	31	27
G	41	46	70	34	18	45	–	25	64
H	41	40	49	47	23	31	25	–	44
I	74	42	26	63	68	27	64	44	–

order of arcs

XE (26)
EG (18)
EH (23)
HF (31)
FB (16)
FC (24)
BD (24)
CI (26)


X
26
E
23
H
18
G
31
F
24
C
26
I
16
B
24
D

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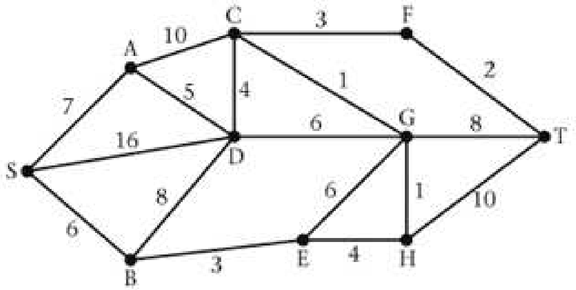
Exercise D, Question 1

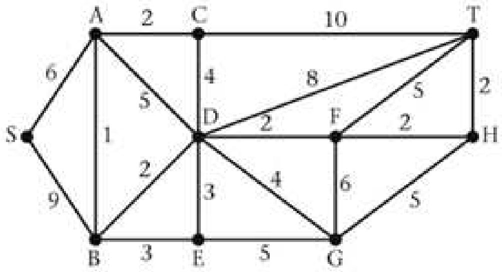
Question:

Use Dijkstra's algorithm to find a shortest route from S to T in each of the following networks. Show your working. State your shortest routes and their lengths. You should show how you obtained your shortest route from your labelled diagrams.

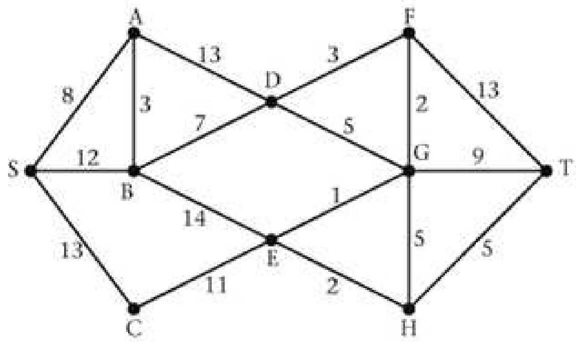
a



b

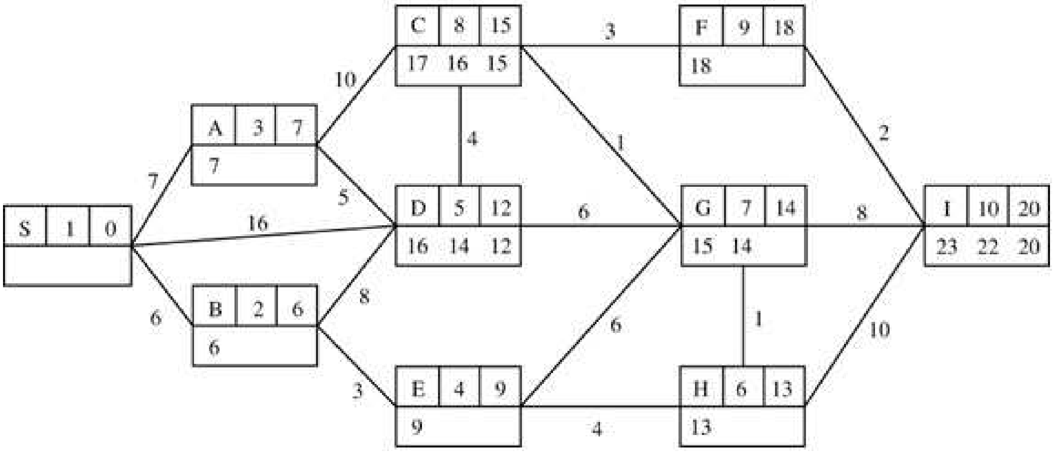


c



Solution:

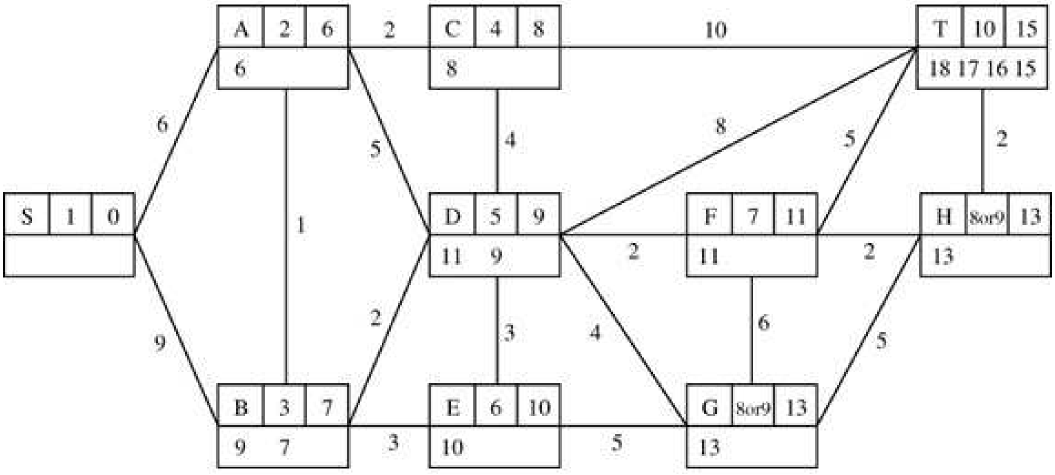
a



20 − 2 = 18 FI
18 − 3 = 15 CF
15 − 1 = 14 GC
14 − 1 = 13 HG
13 − 4 = 9 EH
9 − 3 = 6 BE
6 − 6 = 0 SB

Shortest route: S − B − E − H − G − C − F − T Length of shortest route: 20

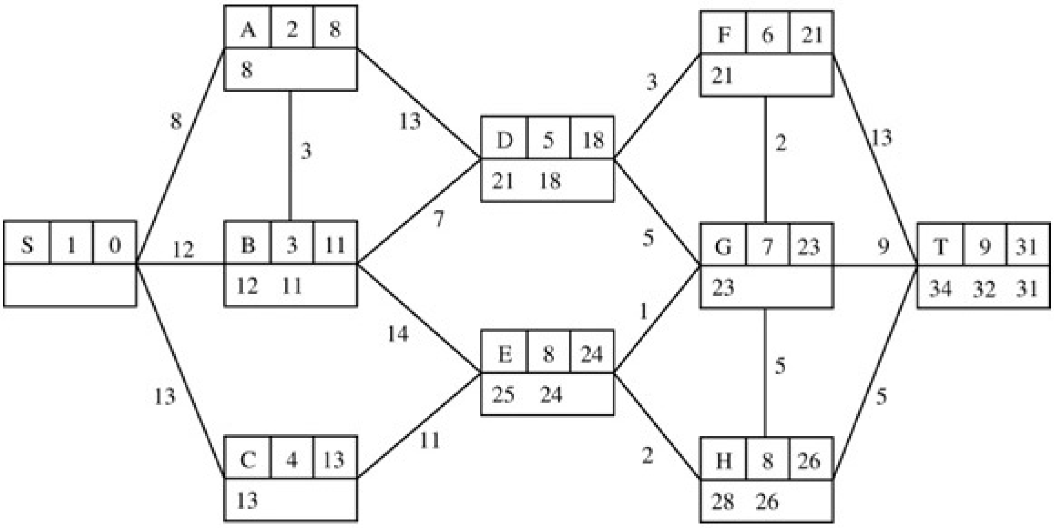
b



15 − 2 = 13 HT
13 − 2 = 11 FH
11 − 2 = 9 DF
9 − 2 = 7 BD
7 − 1 = 6 AB
6 − 6 = 0 SA

Shortest route: S − A − B − D − F − H − T Length of shortest route: 15

c



$31-5=26$ HT
$26-2=24$ EH
$24-11=13$ CE
$13-13=0$ SC
Shortest route: S – C – E – H – T Length of shortest route: 31

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Exercise D, Question 2

Question:

The network shows part of a road network in a city. The network on each arc gives the travel time, in minutes, it takes to travel along that arc.

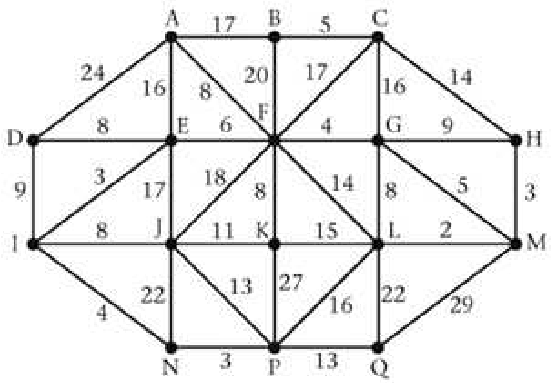
Find

a the quickest route from A to Q and its length,

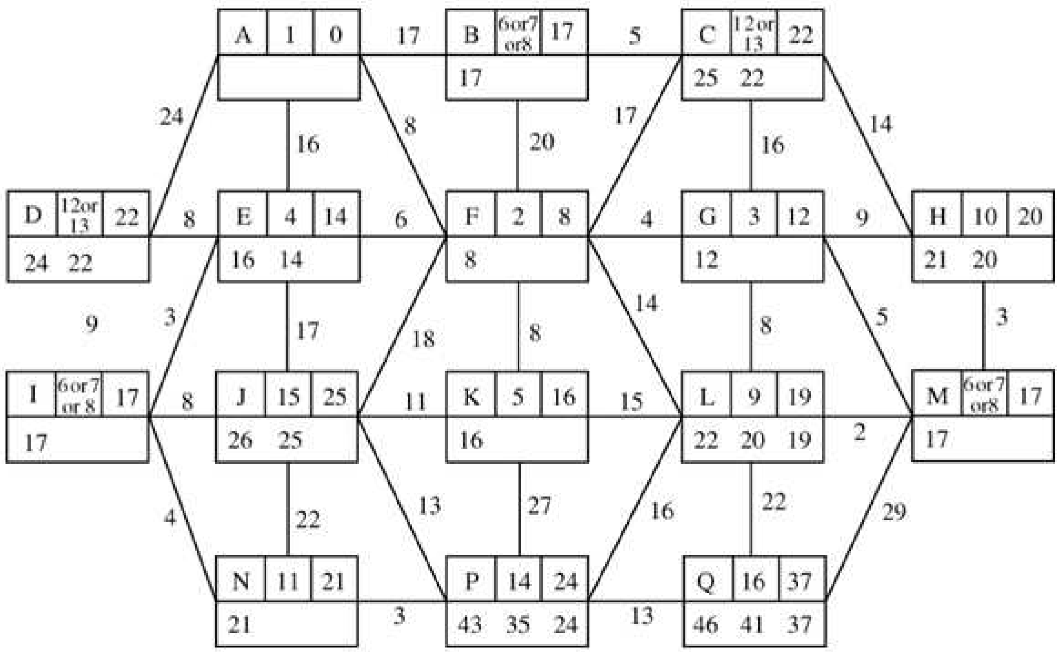
b the quickest route from A to L and its length,

c the quickest route from M to A and its length,

d the quickest route from P to A and its length.



Solution:



a	A to Q	A − F − E − I − N − P − Q	Length	37
b	A to L	A − F − G − M − L	Length	19
c	M to A	M − G − F − A	Length	17
d	P to A	P − N − I − E − F − A	Length	24

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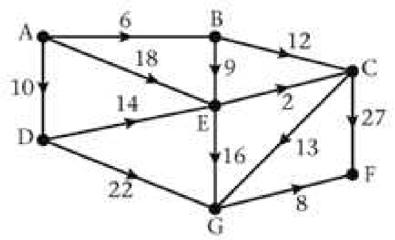
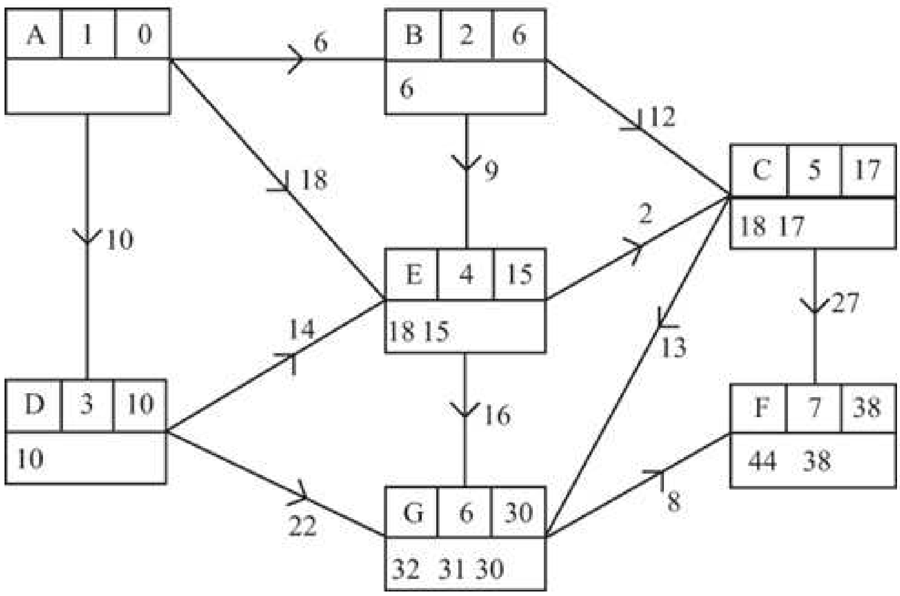
Exercise D, Question 3

Question:

Use Dijkstra's algorithm to find the shortest route, and its length, from A to F in the directed network opposite.



Solution:



Shortest route: A – B – E – C – G – F Length 38

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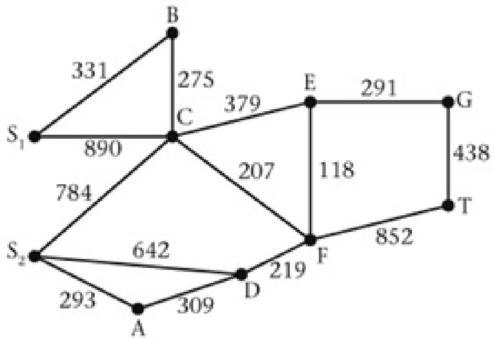
Exercise D, Question 4

Question:

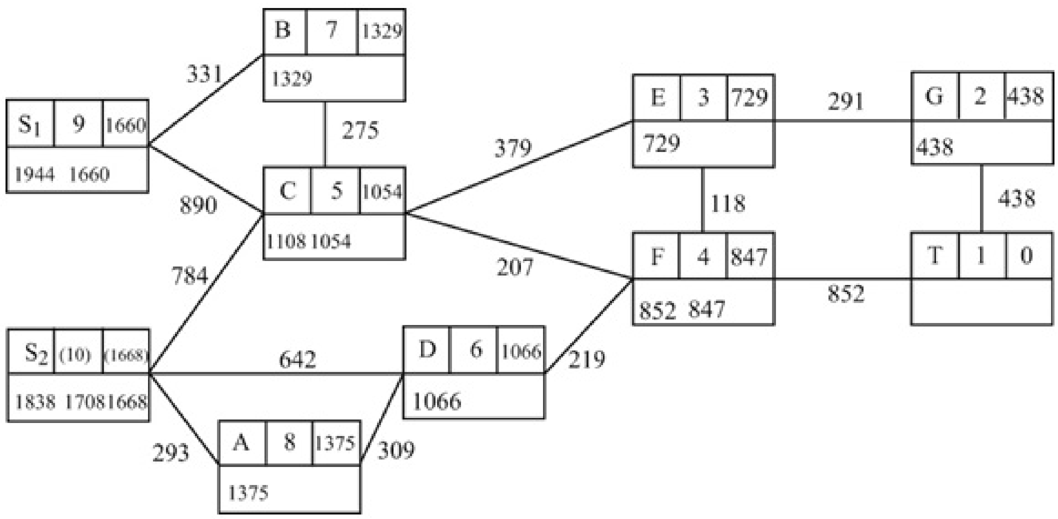
The network represents the distances, in metres, of all roads in a building site. A crane is needed for one day at T. There are two cranes available on site, one at S_1, and the other at S_2. One of these two cranes will be moved to T. In order to minimise the cost it is decided to move the crane that is closest to T. Use Dijkstra's algorithm to determine which crane should be moved.



It is possible to solve this problem with only one application of Dijkstra's algorithm. Think carefully about the starting point.

Solution:

Start at T and work back to S_1 and S_2.



Shortest route $S_1 - B - C - F - E - G - T$

Length of shortest route 1660

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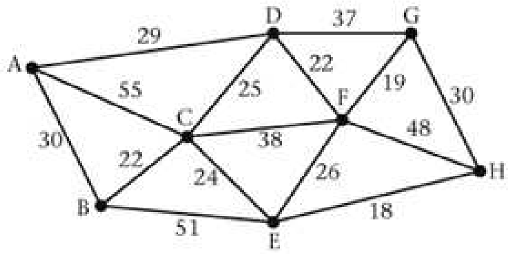
Exercise D, Question 5

Question:

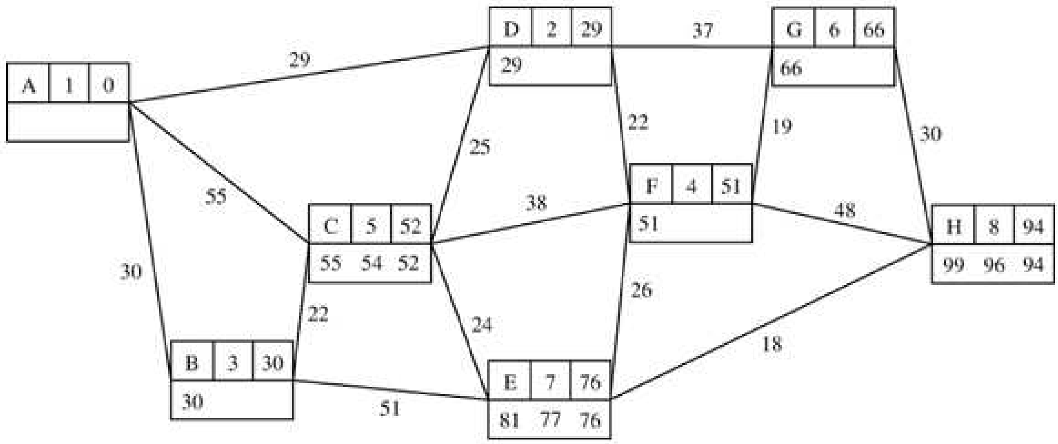


a Use Dijkstra's algorithm to find the shortest route from A to H. Indicate how you obtained your shortest route from your labelled diagram.

b Find the shortest route from A to H via G.

c Find the shortest route from A to H, not using CE.

Solution:



a $94 - 18 = 76$ EH
$76 - 24 = 52$ CE
$52 - 22 = 30$ BC
$30 - 30 = 0$ AB

Shortest route A to H: A – B – C – E – H Length 94

b Shortest route A to H via G: A – D – G – H Length 96

c Shortest route A to H not using CE: A – D – F – E – H Length 95

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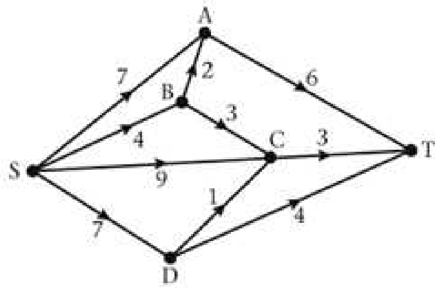
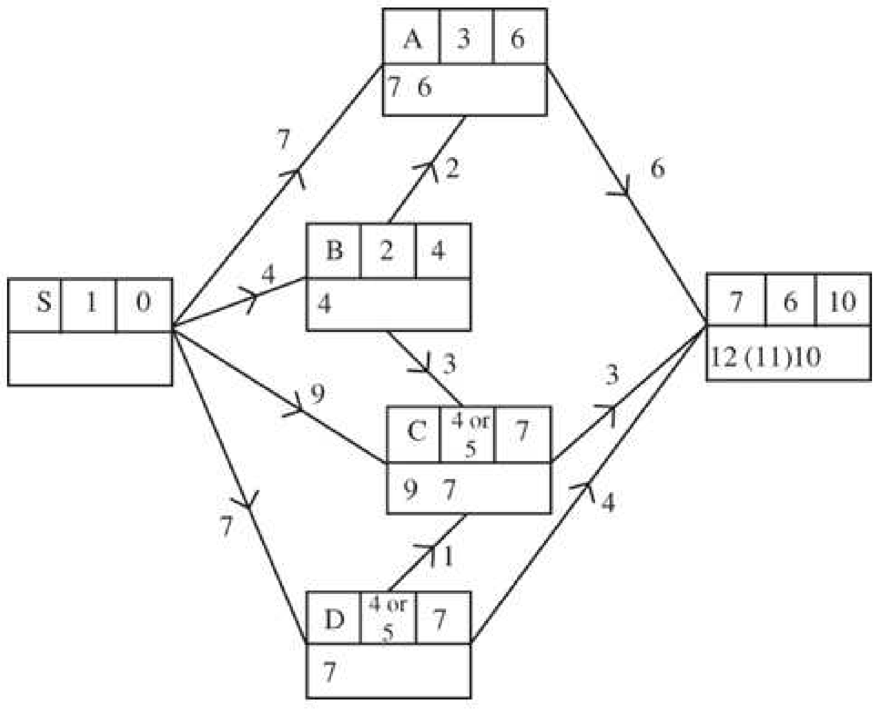
Exercise D, Question 6

Question:



Use Dijkstra's algorithm to find the shortest route from S to T. State the length of your route.

Solution:



Shortest route: S – B – C – T
Length of shortest route: 10

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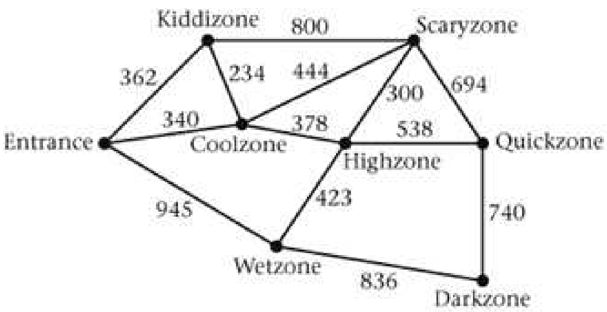
Exercise E, Question 1

Question:

The network represents a theme park with seven zones. The number on each arc shows a distance in metres.



Tramways are to be built to link the seven zones and the car park at the Entrance.

a Find a minimum connector using

i Kruskal's algorithm,

ii Prim's algorithm, starting at the Entrance.

You must make your order of arc selection clear.

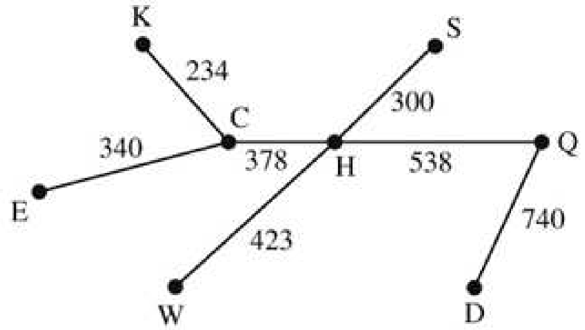
b Draw your tree and state its weight.

Solution:

a **i** Arcs are labeled with initial letters of the nodes.
CK add to tree
SH add to tree
CE add to tree
EK reject
CH add to tree
HW add to tree
CS reject
HQ add to tree
QS reject
QD add to tree
KS reject
DW reject
EW reject

ii EC
CK
CH
HS
HW
HQ
QD

b



weight: 2953

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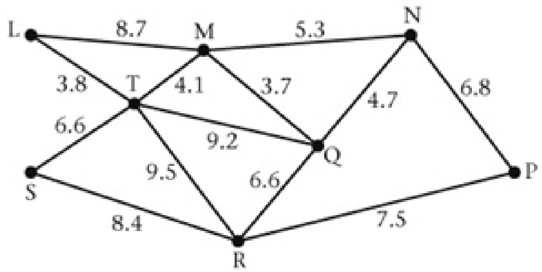
Exercise E, Question 2

Question:

The network represents eight observation points in a wildlife reserve and the possible paths connecting them. The number on each arc is the distance, in kilometres, along that path. It is decided to link the observation points by paths, but in order to minimise the impact on the wildlife reserve, we wish to use the least total length of path.



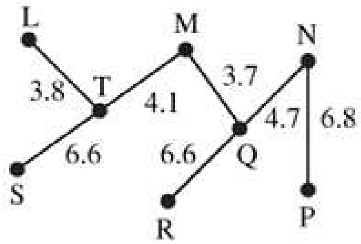
a Find a minimum spanning tree for the network using
  i Prim's algorithm, starting at L,
  ii Kruskal's algorithm,

In each case list the arcs in the order in which you consider them.
Given that paths TQ and RP already exist and so will form part of the tree,

b State which algorithm, Prim's or Kruskal's, you would select to complete the spanning tree. Give a reason for your answer.

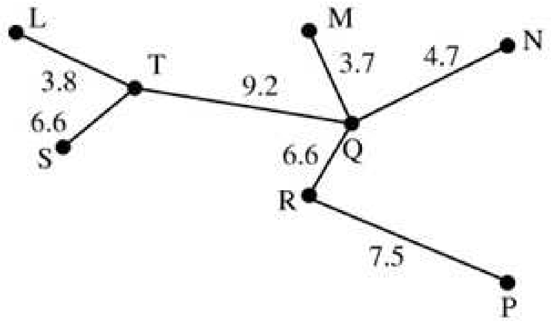
Solution:

a **i** LT
MT
MQ
NQ
ST
QR
NP



ii MQ (3.7) add to tree
LT (3.8) add to tree
MT (4.1) add to tree
NQ (4.7) add to tree
MN (5.3) reject
ST (6.6) add to tree
QR (6.6) add to tree
NP (6.8) add to tree
reject remaining arcs

b Start off the tree with QT and PR then apply Kruskal's algorithm.
Prim's algorithm requires the 'growing' tree to be connected at all times. When using Kruskal's algorithm the tree can be built from non-connected sub-trees.



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Exercise E, Question 3

Question:

	A	B	C	D	E	F
A	–	124	52	87	58	97
B	124	–	114	111	115	84
C	52	114	–	67	103	98
D	87	111	67	–	41	117
E	58	115	103	41	–	121
F	97	84	98	117	121	–

The table shows the distances, in mm, between six nodes A to F in a network.

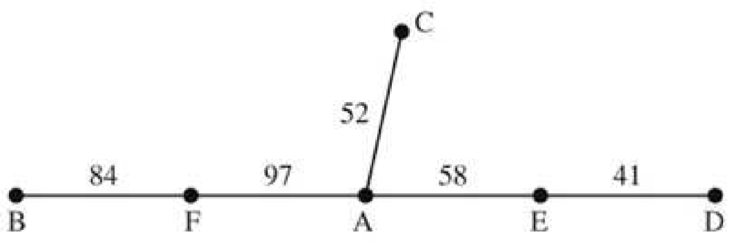
a Use Prim's algorithm, starting at A, to solve the minimum connector problem for this table of distances. You must explain your method carefully and indicate clearly the order in which you selected the arcs.

b Draw a sketch showing the minimum spanning tree and find its length.

Solution:

	↓1	↓6	↓2	↓4	↓3	↓5
	A	B	C	D	E	F
A	–	124	52	87	58	97
B	124	–	114	111	115	84
C	52	114	–	67	103	98
D	87	111	67	–	41	117
E	58	115	103	41	–	121
F	97	84	98	117	121	–

Arcs in order
AC
AE
DE
AF
BF



Length 332 mm

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Exercise E, Question 4

Question:

It is intended to network five computers at a large theme park. There is one computer at the office and one at each of the four different entrances. Cables need to be laid to link the computers. Cable laying is expensive, so a minimum total length of cable is required. The table shows the shortest distances, in metres, between the various sites.

	Office	Entrance 1	Entrance 2	Entrance 3	Entrance 4
Office	–	1514	488	980	945
Entrance 1	1514	–	1724	2446	2125
Entrance 2	488	1724	–	884	587
Entrance 3	980	2446	884	–	523
Entrance 4	945	2125	587	523	–

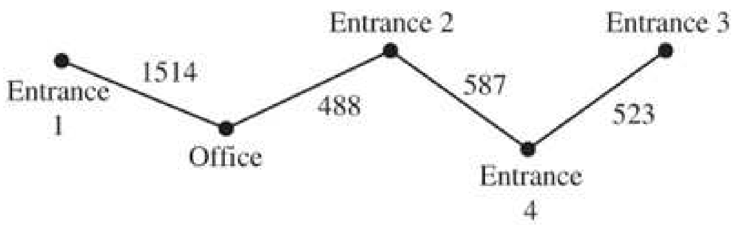
a Starting at Entrance 2, demonstrate the use of Prim's algorithm and hence find a minimum spanning tree. You must make your method clear, indicating the order in which you selected the arcs in your final tree.

b Calculate the minimum total length of cable required.

Solution:

	↓2	↓5	↓1	↓4	↓3
	Office	Entrance 1	Entrance 2	Entrance 3	Entrance 4
Office	–	1514	488	980	945
Entrance 1	1514	–	1724	2446	2125
Entrance 2	488	1724	–	884	587
Entrance 3	980	2446	884	–	523
Entrance 4	945	2125	587	523	–

Arcs in order: Entrance 2–office; Entrance 2–Entrance 4; Entrance 4–Entrance 3;
Office–Entrance 1



Length: 3112 m

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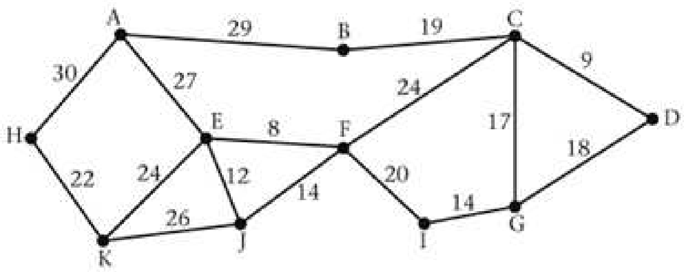
Exercise E, Question 5

Question:



You are to use Kruskal's algorithm to find a minimum spanning tree for the network shown.

a **i** Write down the order in which you selected the arcs.

ii Sketch your minimum spanning tree.

iii State the weight of your minimum spanning tree.

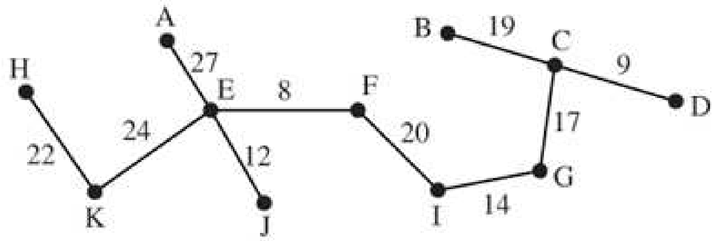
For any connected network,
E = the number of edges in the minimum spanning tree, and
V = the number of vertices in the network.

b Write down the relationship between E and V.

Solution:

a **i** Order of arcs
EF (8) add to tree
CD (9) add to tree
EJ (12) add to tree
FJ (14) reject
GI (14) add to tree
CG (17) add to tree
DG (18) reject
BC (19) add to tree
FI (20) add to tree
HK (22) add to tree
EK (24) add to tree
CF (24) reject
JK (26) reject
AE (27) add to tree
AB (29)
AH (30) reject remaining arcs

ii



iii weight: 172

b $V = E + 1$

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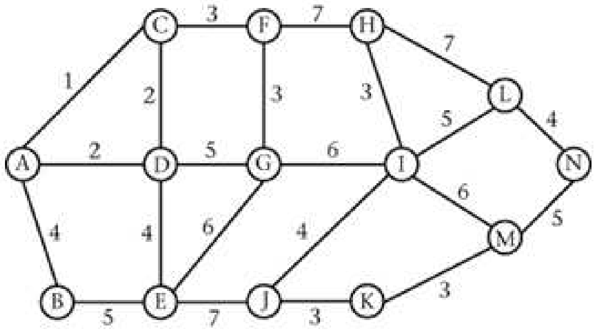
Exercise E, Question 6

Question:

A company is to install power lines to buildings on a large industrial estate. The lines are to be laid by the side of the roads on the estate. The estate is shown as a network opposite. The buildings are designated A, B, C, …, N and the distances between them are given in hundreds of metres. The manager wants to minimise the total length of power line to be used.



a Use Kruskal's algorithm to obtain a minimum spanning tree for the network and hence determine the minimum length of power line needed.

Owing to a change of circumstances, the company modifies its plans for the estate. The result is that the road from F to G now has a length of 700 metres.

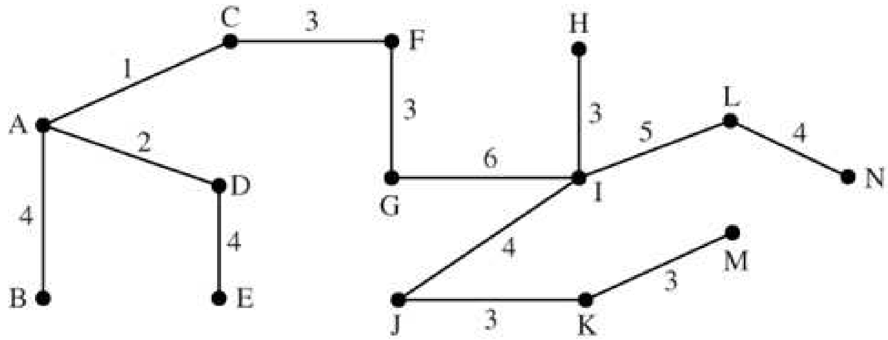
b Determine the new minimum total length of power line.

Solution:

a Order of arcs

AC (1) add to tree
AD (2) add to tree
CD (2) reject
CF (3) add to tree
FG (3) add to tree
HI (3) add to tree
KM (3) add to tree
JK (3) add to tree
AB (4) add to tree
DE (4) add to tree
IJ (4) add to tree
LN (4) add to tree
DG (5) reject
BE (5) reject
IL (5) add to tree
MN (5) reject
EG (6) reject
GI (6) add to tree
IM (6)
FH (7)
HL (7)
EJ (7)
reject remaining arcs

weight = 45 so 4500 m needed



b Remove FG (7) and replace with DG (5) weight = 47 so 4700 m

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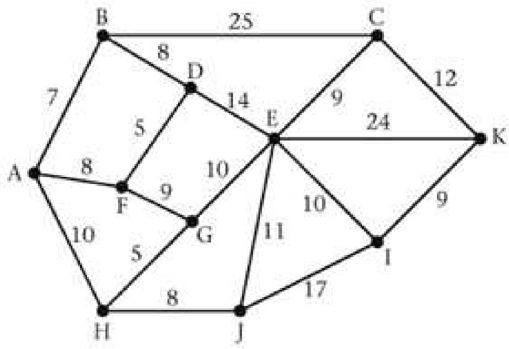
Exercise E, Question 7

Question:



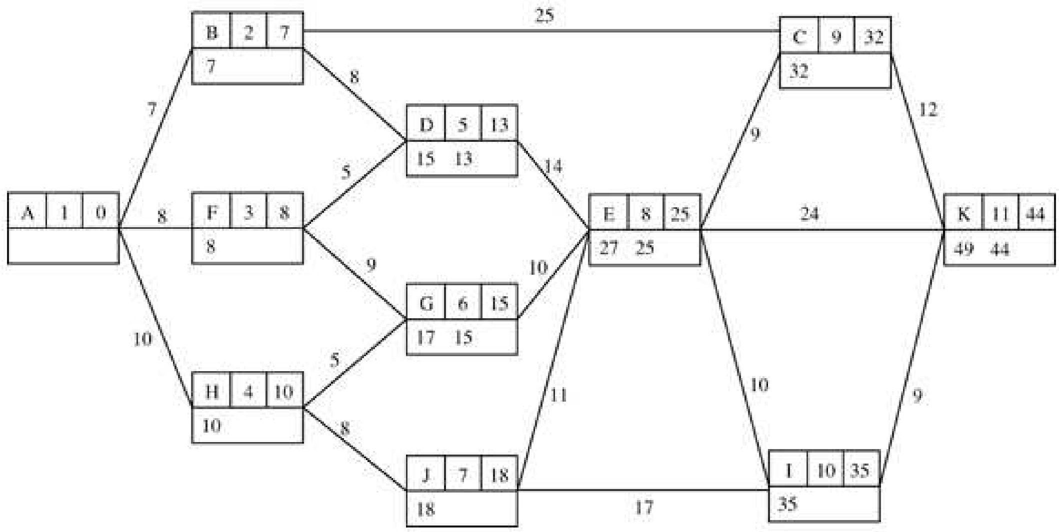
A weighted network is shown above. The number on each arc indicates the weight of that arc.

a Use Dijkstra's algorithm to find a path of least weight from A to K.
State clearly
i the order in which the vertices were labelled.
ii how you determined the path of least weight from your labelling.

b List all alternative paths of least weight.

c Describe a practical problem that could be modelled by the above network and solved using Dijkstra's algorithm.

Solution:



a **i** Possible paths are A–H–G–E–I–K
and A–H–J–I–K
and A–B–C–K

Any one accepted

ii

$44-9=35$	IK		$44-9=35$	IK		$44-12=32$	CK
$35-10=25$	EI		$35-17=18$	JI		$32-25=7$	BC
$25-10=15$	GE	or	$18-8=10$	MJ	or	$7-7=0$	AB
$15-5=10$	HG		$10-10=0$	AH			
$10-10=0$	AH						

b A–H–G–E–I–K and A–H–J–I–K and A–B–C–K

c The arcs could be roads.
The nodes could be junctions
The number on each arc could be the distance in km.

The network, together with Dijkstra's algorithm,
could be used to find the shortest route from A to K.

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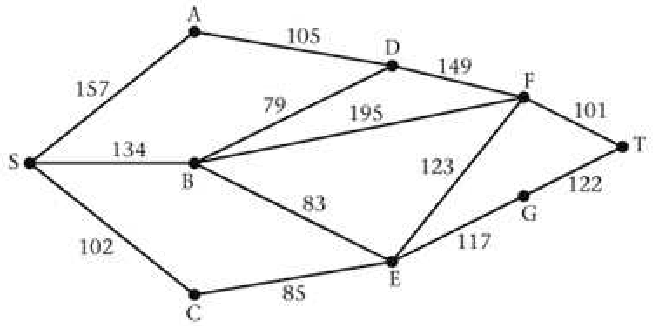
Exercise E, Question 8

Question:



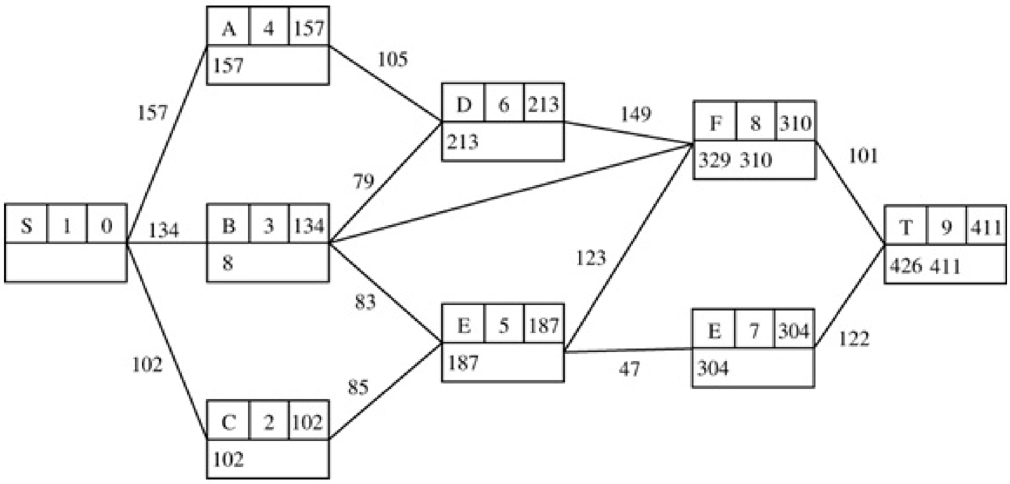
The network above shows the distances, in miles, between nine cities. Use Dijkstra's algorithm to determine the shortest route, and its length, between cities S and T. You must indicate clearly

i the order in which the vertices are labelled,

ii how you used your labelled diagram to decide which cities to include in the shortest route.

Solution:

a



Order of vertex labelling: S C B A E D G F T

Route : S–C–E–F–T

$411-101=310$ FT

$310-123=187$ EF

$187-85=102$ CE

$102-102=0$ SC

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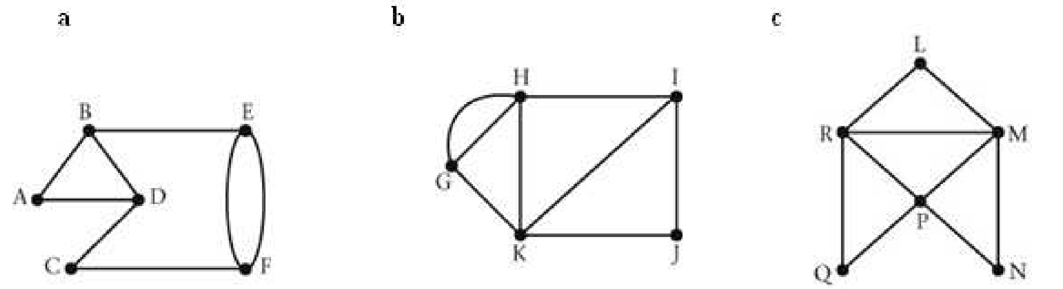
Exercise A, Question 1

Question:

List the valency of each vertex and hence determine if each of the graphs below are

- **i** Eulerian
- **ii** semi-Eulerian
- **iii** neither.

For those that are Eulerian or semi-Eulerian, find a route that traverses each arc just once.



Solution:

a

vertex	A	B	C	D	E	F
valency	2	3	2	3	3	3

There are 4 nodes with odd valency so the graph is *neither* Eulerian nor Semi-Eulerian.

b

vertex	G	H	I	J	K
valency	3	4	3	2	4

There are precisely 2 nodes of odd degree (G and I) so the graph is *semi-Eulerian*.
A possible route starting at G and finishing at I is:
G – H – K – I – J – K – G – H – I.

c

vertex	L	M	N	P	Q	R
valency	2	4	2	4	2	4

All vertices have even valency, so the graph is *Eulerian*.
A possible route starting and finishing at L is:
L – M – N – P – M – R – P – Q – R – L.

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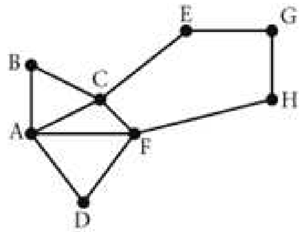
Exercise A, Question 2

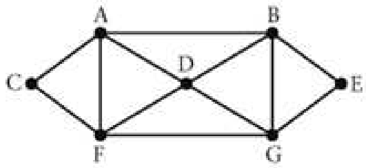
Question:

a Show that each of the graphs below is Eulerian.
b In each case, find a route that starts and finishes at A and traverses each arc just once.

i



ii



Solution:

a **i**

vertex	A	B	C	D	E	F	G	H
valency	4	2	4	2	2	4	2	2

ii

vertex	A	B	C	D	E	F	G
valency	4	4	2	4	2	4	4

b **i**

A possible route is: A – B – C – A – F – C – E – G – H – F – D – A.

ii

A possible route is: A – C – F – A – B – E – G – B – D – G – F – D – A.

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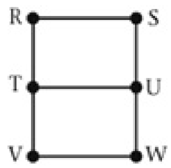
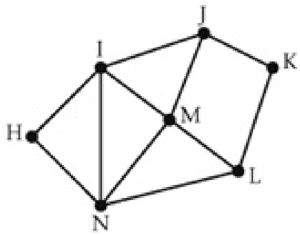
Exercise A, Question 3

Question:

i



ii



a Show that each of these graphs is semi-Eulerian.

b In each case, find a route, starting and finishing at different vertices, that traverses each edge just once.

Solution:

a i

vertex	R	S	T	U	V	W
valency	2	2	3	3	2	2

Precisely 2 vertices of odd valency (T and U) so semi-Eulerian.

ii

vertex	H	I	J	K	L	M	N
valency	2	4	3	2	3	4	4

Precisely 2 nodes of odd degree (J and L) so semi-Eulerian.

b i A possible route starting of T and finishing at U is:
T – R – S – U – W – V – T – U.

ii A possible route starts at J and finishes at L:
J – K – L – M – J – I – M – N – I – H – N – L.

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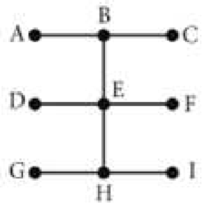
Exercise A, Question 4

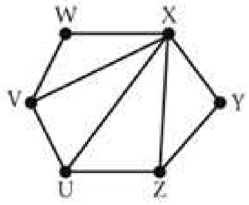
Question:

Explain why each of the graphs below is not traversable.

a



b



Solution:

a

vertex	A	B	C	D	E	F	G	H	I
valency	1	3	1	1	4	1	1	3	1

There are more than 2 vertices of odd degree so the graph is not traversable.

b

vertex	U	V	W	X	Y	Z
valency	3	3	2	5	2	3

There are more than 2 nodes of odd degree so the graph is not traversable.

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Exercise A, Question 5

Question:

Considering the valencies of each vertex in Questions **1** to **4**, verify the hand-shaking lemma.

Solution:

In each case there are either zero, or an even number of, vertices with odd valency.

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Exercise A, Question 6

Question:

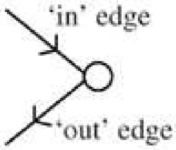
Explain why a traversable graph has either

a all of its vertices with even valency or

b precisely two vertices of odd valency, these being the start and finish points.

Solution:

If a graph is traversable we will approach each vertex on one edge and must leave on a different one.



a This means that the edges must be in pairs at each vertex, an 'out' edge for each 'in' edge, and since the graph is traversable there will be no edges left over. So the valency of each vertex will be even if we return to the start.

b For routes that start and finish at different vertices there will be an unpaired 'out' edge from the start vertex which will be balanced by an unpaired 'in' edge at the finish vertex. So these two vertices will have odd valency, but all others will be even.

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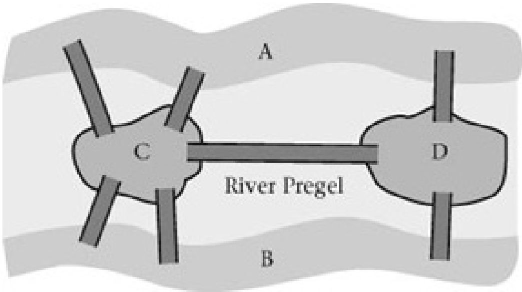
Exercise A, Question 7

Question:

The diagram represents the city of Königsberg (Prussia, now Kaliningrad, Russia). The Pregel river runs through the city and creates two large islands in the centre. The two islands (C and D) were linked to each other and the mainland (A and B) by seven bridges.



The problem for the citizens of Königsberg was to decide whether or not it was possible to walk a route that crossed each bridge just once and returned to its starting point.

This is the famous 'Bridges of Königsberg' problem.

a Using four vertices A, B, C and D to represent the four parts of the city, and seven arcs to represent the bridges, draw a graph to model the problem.

b Show that the graph is not traversable.

There is a continuation of the problem. Johannes works at A, Gregor works at B and Peter works at D. There is a hotel at C.

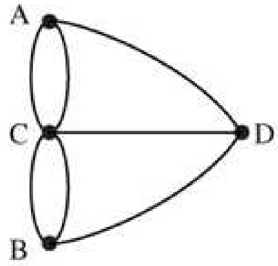
c Johannes builds an eighth bridge so that he can start at A and finish at his home at C, crossing each bridge once. However, he does not want Gregor to be able to find a similar route from B to C. Where should Johannes build his eighth bridge?

d Gregor decides to build a ninth bridge so that he can start at B and finish at his home near C, crossing each bridge once. He does not want Johannes to be able to find a similar route from A to C. Where should Gregor build his ninth bridge?

e Peter decides to build a tenth bridge, so that every person in the city can cross all the bridges in turn and return to their starting point. Where should Peter build the tenth bridge?

Solution:

a

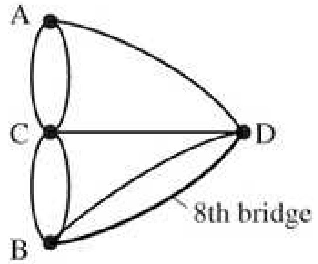


b

vertex	A	B	C	D
valency	3	3	5	3

There are more than two odd nodes, so the graph is *not* traversable.

c

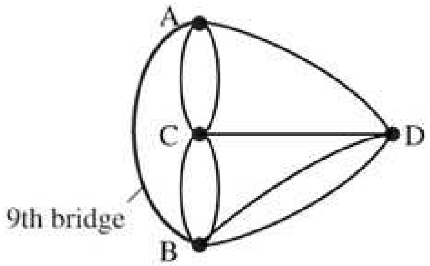


We will start of A and finish at C so these still need to have odd valency. We can only have two odd valencies so B and D must have even valencies (see table).

We need to change the valency of B and of D. So we build a bridge from B to D.

vertex	A	B	C	D
valency with 7 bridges	odd	*odd*	odd	*odd*
valency wanted	odd	*even*	odd	*even*

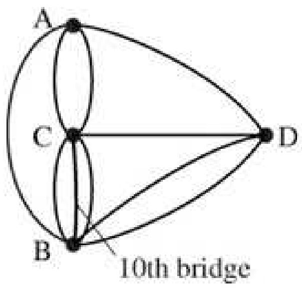
d



We will start at B and finish at C so these vertices need to be the two vertices with odd valency. We need A and D to have even valency (see table).
We need to change the valency of node A and of node B.
So we build a bridge from A to B.

vertex	A	B	C	D
valency with 8 bridges	*odd*	*even*	odd	even
valency wanted	*even*	*odd*	odd	even

e



All vertices now need to have even valency.
This means we need to change the valencies of nodes B and C.
So the 10th bridge needs to be built from B to C.

vertex	A	B	C	D
valency with 9 bridges	even	*odd*	*odd*	even
valency wanted	even	*even*	*even*	even

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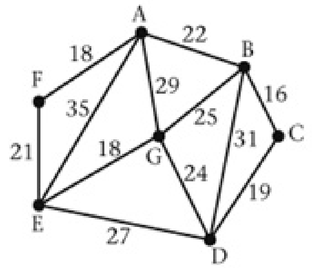
Exercise B, Question 1

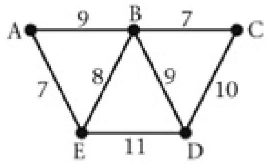
Question:

Solve the route inspection problem for each of the networks below. In each case, state your minimal route and its length.

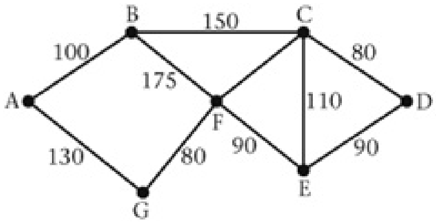
a



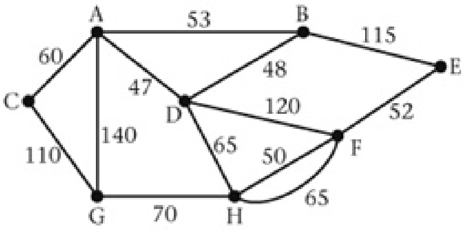
b



c

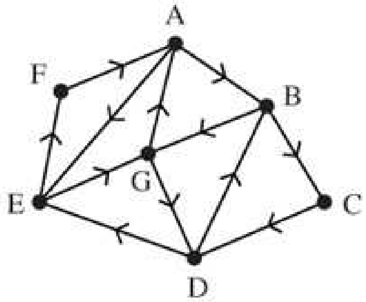


d



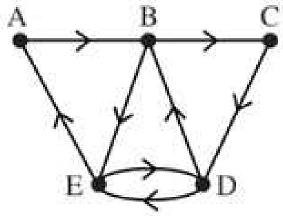
Solution:

a All valencies are even, so the network is traversable and can return to its start.



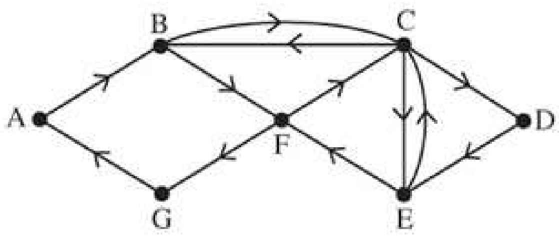
A possible route is:
A – B – C – D – B – G – D – E – G – A – E – F – A.
length of route = weight of network
= 285

b The valencies of D and E are odd, the rest are even.
We must repeat the shortest path between D and E, which is the direct path DE.
We add this extra arc to the diagram.



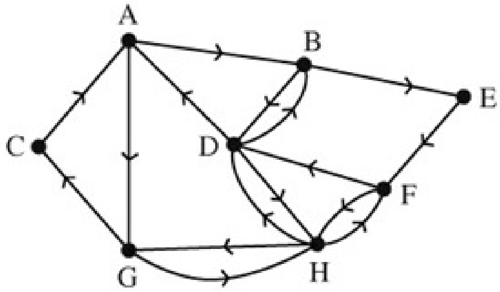
A possible route is:
A – B – C – D – E – D – B – E – A.
length of route = weight of network + arc DE
= 61 + 11
= 72

c The degrees of B and E are odd, the rest are even.
We must repeat the shortest path from B to E.
By inspection this is BCE, length 260.
We add these extra arcs to the diagram.



A possible route is:
A – B – C – D – E – C – B – F – C – E – F – G – A.
length of route = weight of network + BCE
= 1055 + 260
= 1315

d The order of B and G are odd, the rest are even.
We must repeat the shortest path from B to G.
By inspection this is BDHG, length 183.
We add these arcs to the diagram.



A possible route is:
A − B − E − F − D − B − D − A − G − H − F − H − D − H − G − C − A.

$$\begin{aligned}\text{length of route} &= \text{weight of network} + \text{BDHG}\\ &= 995 + 183\\ &= 1178\end{aligned}$$

Solutionbank D1

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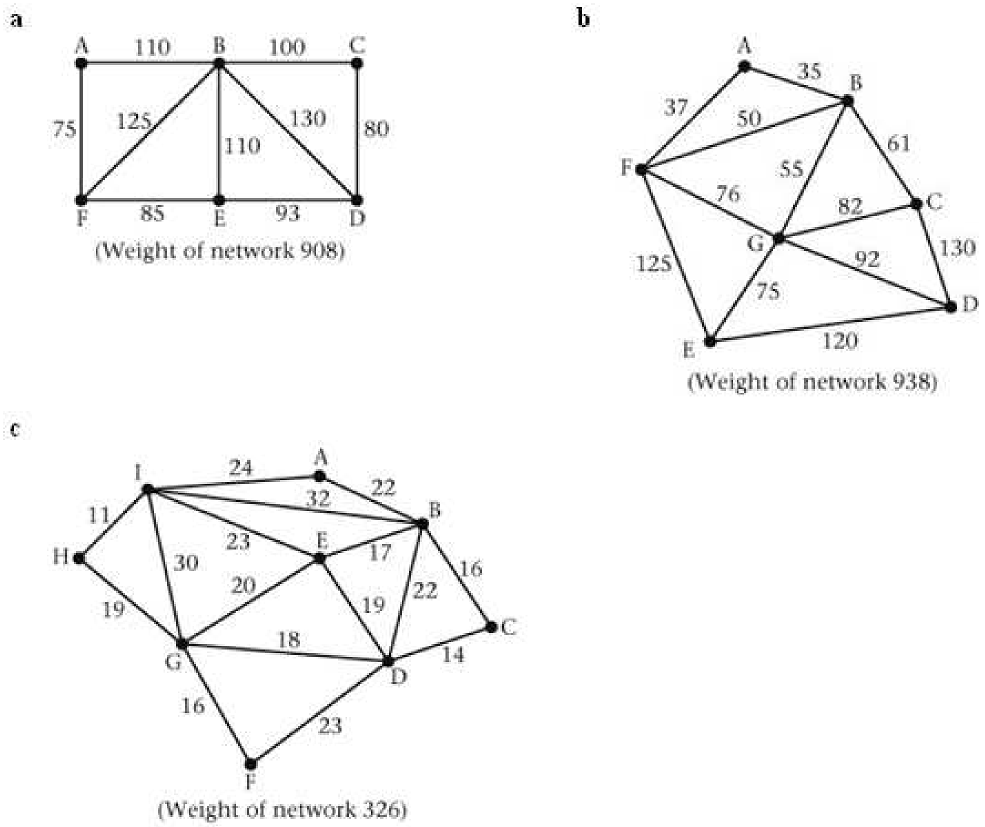
Exercise B, Question 2

Question:

Each of the diagrams below show a network of roads that need to be inspected. In each case, find the length of the shortest route that traverses each arc at least once and returns to the start vertex. State your routes.



Solution:

a Odd valencies at B, D, E and F.
Considering all possible pairings and their weights.

$BD + EF = 130 + 85 = 215 \leftarrow$ least weight

$BE + DF = 110 + 178 = 288$

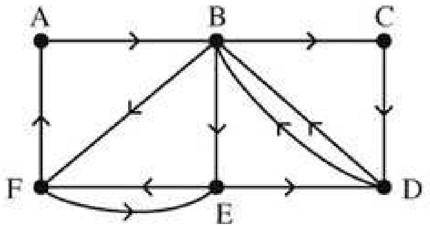
Shortest route D to F, is DEF.

$BF + DE = 125 + 93 = 218$

We need to repeat arcs BD and EF.

$$\begin{aligned}\text{The length of the shortest route} &= \text{weight of network} + 215\\ &= 908 + 215\\ &= 1123\end{aligned}$$

Adding BD and EF to the diagram gives.



A possible route is:
$A - B - C - D - B - E - D - B - F - E - F - A$.

b Odd valencies at C, D, E and G
Considering all possible pairings and their weights

$CD + EG = 130 + 75 = 205$

$CE + DG = 157 + 92 = 249$

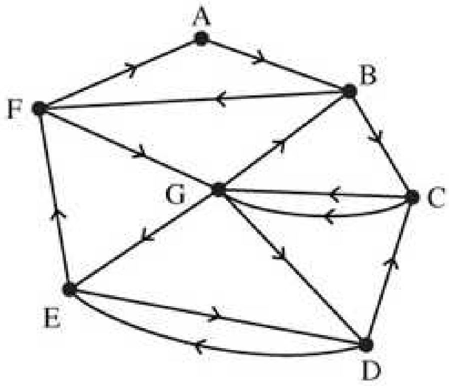
Shortest route from C to E is CGE.

$CG + DE = 82 + 120 = 202 \leftarrow$ least weight

We need to repeat arcs CG and DE.

$$\begin{aligned}\text{The length of the shortest route} &= \text{weight of network} + 202\\ &= 938 + 202\\ &= 1140\end{aligned}$$

Adding CG and DE to the diagram gives.



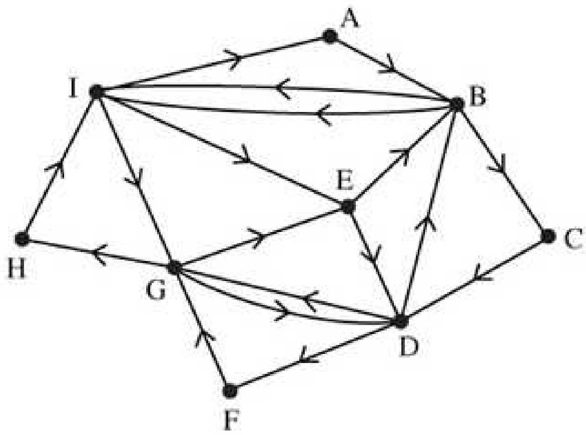
A possible route is:
A – B – C – G – D – C – G – E – D – E – F – G – B – F – A.

c Odd degrees at B, D, G and I.
Considering all possible pairings and their weight
BD + GI = 22 + 30 = 52
BG + DI = 37 + 42 = 79
BI + DG = 32 + 18 = 50 ← least weight
We need to repeat arcs BI and DG.

Shortest routes
BG = BEG
DI = DEI

The length of the shortest route = weight of network + 50
= 326 + 50
= 376

Adding BI and DG to the diagram gives.



A possible route is
A – B – C – D – B – I – E – B – I – G – D – F – G – E – D – G – H – I – A.

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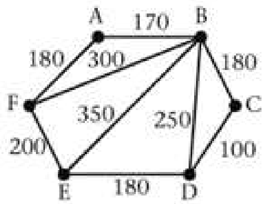
Exercise B, Question 3

Question:

The diagram shows the paths in a park. The number on each arc gives the length, in metres, of that path. The vertices show the park entrances, A, B, C, D, E and F.



A gardener needs to inspect each path for weeds.
She will walk along each path once and wishes to minimise her route.

a Use the route inspection algorithm to find a minimum route, starting and finishing at entrance A. State the length of your route.

Given that it is now permitted to start and finish at two different entrances,

b find the start and finish points that would give shortest route, and state the length of the route.

Solution:

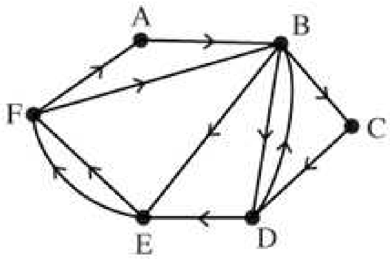
a Odd degrees are B, D, E and F.
Considering all possible pairings and their weight

$BD + EF = 250 + 200 = 450 \leftarrow$ least weight

$BE + DF = 350 + 380 = 730$

$BF + DE = 300 + 180 = 480$

We need to repeat arcs BD and EF.
Adding these to the diagram gives



A possible route is:
A − B − C − D − B − D − E − F − B − E − F − A.
length=1910+450=2360.

b We will still have two odd valencies.
We need to select the pair that gives the least path.
From part a our six choices are
BD (250), EF (200), BE (350), DF (380), BF (300) and DE (180).
The shortest is DE (180) so we choose to repeat this.
It is the other two vertices (B and F) that will be our start and finish.
For example, start at B, finish at F
length of route $= 1910 + 180 = 2090$

Solutionbank D1
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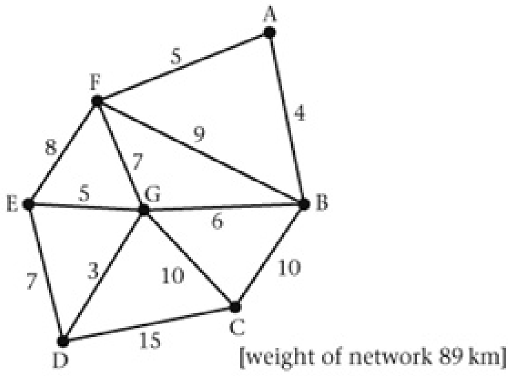
Exercise B, Question 4

Question:

The diagram represents a system of roads.



The number on each arc gives the distance, in kilometres, of that road.
The town council need to renew the road markings.
Cherry will be renewing the kerbside markings and Mac will renew the centre road markings.

Cherry needs to travel along each road twice, once on each side of the road.

a Explain how this differs from the standard route inspection problem and find the length of Cherry's route.

Mac must travel along each road once.
b Use the route inspection algorithm to find a minimal route. You should state the roads he will traverse twice and the length of his route.

Road EG is being resurfaced soon and it is decided not to renew its road markings until after the resurfacing.

Given that EG may be omitted from his route,
c find the length of Mac's minimal route.

Solution:

a Each arc must be traversed twice, whereas in the standard problem each arc need only be visited once.
This has the same effect as doubling up all the edges
The length of the route $= 2 \times$ weight of network

$$= 2 \times 89 = 178 \text{ km}$$

b Odd nodes C, D, E, G.
Consider all possible pairings.

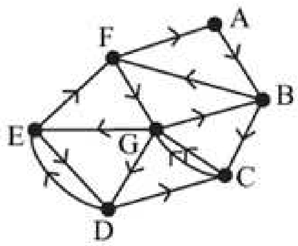
$CD + EG = 13 + 5 = 18$

$CE + DG = 15 + 3 = 18$

$CG + DE = 10 + 7 = 17 \leftarrow$ least weight

Shortest routes
CD is CGD
CE is CGE

We need to repeat arcs CG and DE.
Adding these to the network



A possible route is:
$A - B - C - G - D - C - G - E - D - E - F - G - B - F - A.$
Length $= 89 + 17 = 106$ km

c If EG is omitted E and G become even and the only odd valencies are at C and D.
We must repeat the shortest path between C and D, CGD.
The new length $= 89 + 13 = 102$ km

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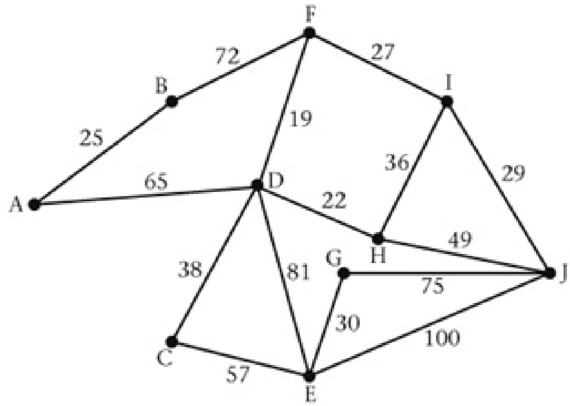
Exercise C, Question 1

Question:

The network of paths in a garden is shown below. The numbers on the paths give their lengths in metres. The gardener wishes to inspect each of the paths to check for broken paving slabs so that they can be repaired before the garden is opened to the public. The gardener has to walk along each of the paths at least once.



a Write down the degree (valency) of each of the ten vertices $A, B, \ldots, J$.

b Hence find a route of minimum length. You should clearly state, with reasons, which, if any, paths will be covered twice.

c State the total length of your shortest route. ***E***

Solution:

a

vertex	A	B	C	D	E	F	G	H	I	J
degree	2	2	2	5	4	3	2	3	3	4

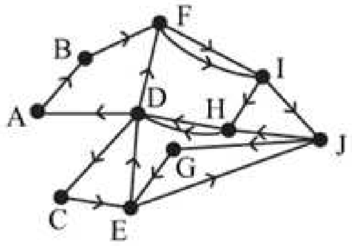
b $DF + HI = 19 + 36 = 55$

$DH + FI = 22 + 27 = 49 \leftarrow$ least weight

$DI + HF = 46 + 41 = 87$

Shortest routes
D to I is DFI
H to F is HDF

Repeat DH and FI
Add these to the network to get



A possible route is
A – B – F – I – J – G – E – J – H – D – F – I – H – D – C – E – D – A.

c length $= 725 + 49 = 774$

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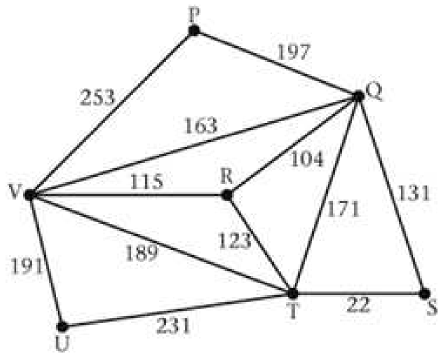
Exercise C, Question 2

Question:



Starting and finishing at P, solve the route inspection (Chinese postman) problem for the network shown above. You must make your method and working clear.
State:

a your route, using vertices to describe the arcs

b the total length of your route. ***E***

Solution:

Odd vertices Q, R, T, V.
Considering all possible pairings and their weight

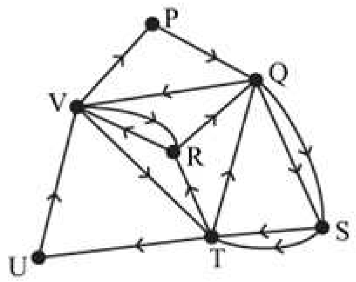
Shortest route from Q to T is QST.

$QR + TV = 104 + 189 = 293$

$QT + RV = 153 + 115 = 268 \leftarrow$ least weight

$QV + RT = 163 + 123 = 286$

Repeat arcs QS, ST and RV.
Add these to the network



A possible route is
P − Q − S − T − Q − S − T − R − Q − V − R − V − T − U − V − P.
length of route $= 1890 + 268$
$= 2158$

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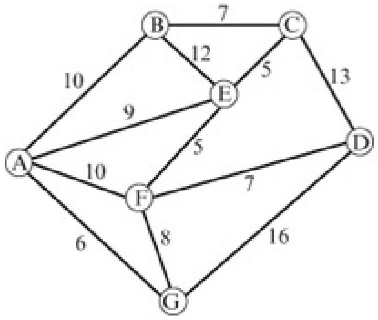
Exercise C, Question 3

Question:



The diagram shows the network of paths in a garden to be opened to the public. The number on each path gives its length in metres. The gardener wishes to inspect each of the paths to check for broken paving slabs, so that they can be repaired before the garden is opened.

a Write down the degree (valency) of the seven vertices A, B, C, D, E, F and G.

b Use an appropriate algorithm to find a route of minimum length which starts and finishes at A and which traverses each path at least once. Write down which paths, if any, will be traversed twice.

c Calculate the total length of your shortest route. ***E***

Solution:

a

vertex	A	B	C	D	E	F	G
degree	4	3	3	3	4	4	3

Odd valencies at B, C, D and G.

b Considering all possible pairings and their weight

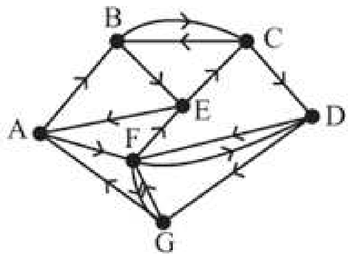
$BC + DG = 7 + 15 \text{ via } F = 22$

$BD + CG = 20 + 18 = 38$

$BG + CD = 16 + 13 = 29$

BC, DF and FG should be repeated.
Adding these arcs to the network gives

Shortest routes
DG is DFG
BD is BCD
CG is CEFG
BG is BAG



A possible route is
A – B – C – B – E – C – D – F – D – G – F – G – A – F – E – A.

c Length $= 108 + 22 = 130$

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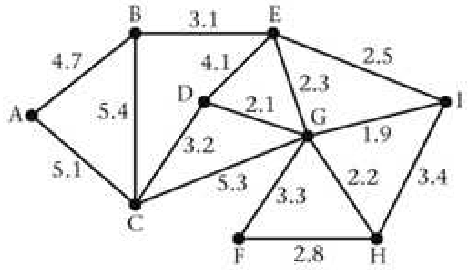
Exercise C, Question 4

Question:

The network shows the major roads that are to be gritted by a council in bad weather. The number on each arc is the length of the road in kilometres.



a List the valency of each of the vertices.

b Starting and finishing at A, use an algorithm to find a route of minimum length that covers each road at least once. You should clearly state, with reasons, which (if any) roads will be traversed twice.

c Obtain the total length of your shortest route.

There is a minor road BD (not shown) between B and D of length 6.4 km. It is not a major road so it does not need gritting urgently.

d Decide whether or not it is sensible to include BD as a part of the main gritting route, giving your reasons. (You may ignore the cost of the grit.) ***E***

Solution:

a

vertex	A	B	C	D	E	F	G	H	I
valency	2	3	4	3	4	2	6	3	3

Odd valencies at B, D, H and I.

b Considering all possible complete pairings and their weight

$BD + HI = 7.2 + 3.4 = 10.6$

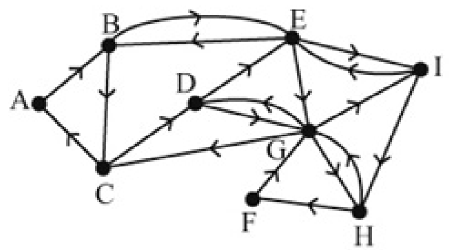
$BH + DI = 7.6 + 4 = 11.6$

$BI + DH = 5.6 + 4.3 = 9.9 \leftarrow$ least weight

Repeat BE, EI and DG, GH.

Adding these arcs to the network gives

Shortest routes
BD is BED
BH is BEGH
DI is DGI
BI is BEI
DH is DGH



A possible route is:

A – B – E – I – H – G – I – E – B – C – D – G – D – E – G – H – F – G – C – A.

c length $= 51.4 + 9.9 = 61.3$ km

d If BD is included B and D now have even valency.

Only H and I have odd valency.

So the shortest path from H to I needs to be repeated.

$$\begin{aligned}\text{Length of new route} &= 51.4 + BD + \text{path from H to I} \\ &= 51.4 + 6.4 + 3.4 \\ &= 61.2 \text{ km}\end{aligned}$$

This is (slightly) shorter than the previous route so choose to grit BD since it saves 0.1 km.

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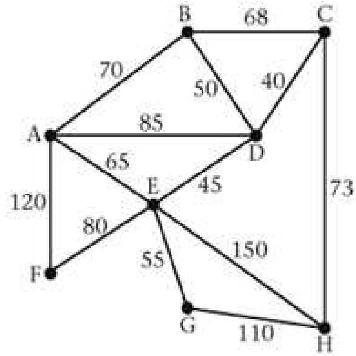
Exercise C, Question 5

Question:

The network opposite represents the streets in a village. The number on each arc represents the length of the street in metres.

The junctions have been labelled A, B, C, D, E, F, G and H.

An aerial photographer has taken photographs of the houses in the village. A salesman visits each house to see if the occupants would like to buy a photograph of their house. He needs to travel along each street at least once. He parks his car at A and starts and finishes there. He wishes to minimise the total distance he has to walk.



a Describe an appropriate algorithm that can be used to find the minimum distance the salesman needs to walk.

b Apply the algorithm and hence find a route that the salesman could take, stating the total distance he has to walk.

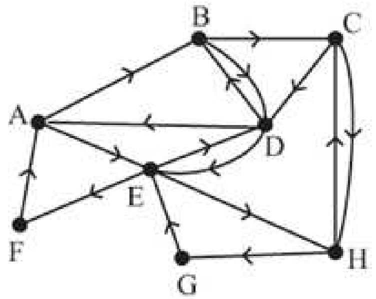
c A friend offers to drive the salesman to B at the start of the day and collect him from C later in the day.
Explaining your reasoning, carefully determine whether this would increase or decrease the total distance the salesman has to walk. ***E***

Solution:

a The route inspection algorithm
(method as shown in main text page 69)

b Odd valencies B, C, E, H.
Considering all possible complete pairings and their weight
$BC + EH = 68 + 150 = 218$
$BE + CH = 95 + 73 = 168 \leftarrow$ least weight
$BH + CE = 141 + 85 = 226$
Repeat BD, DE and CH
Adding these arcs to the network gives

Shortest routes
BE is BDE
BH is BCH, CE is CDE



A possible route is:
$A - B - D - B - C - H - C - D - E - D - A - E - H - G - E - F - A.$
length $= 1011 + 168$
$= 1179$ m

c This would make B the start and C the finish.
We would have to repeat the shortest path between E and H only.
New route $= 1011 + 150 = 1161$ m .
$1161 < 1179$
So this would decrease the total distance by 18 m.

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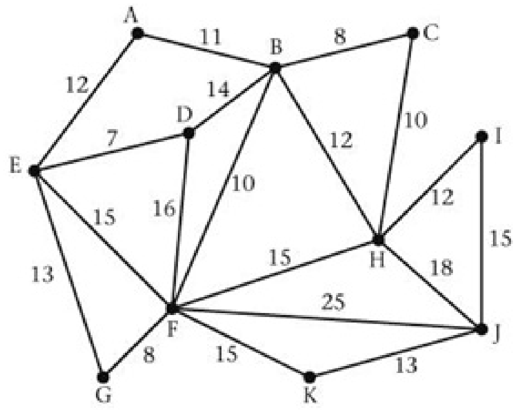
Exercise C, Question 6

Question:



a Describe an algorithm that is used to solve the route inspection (Chinese postman) problem.

b Apply an algorithm and find a route, starting and finishing at A, that solves the route inspection problem for the network shown.

c State the total length of your route.

The situation is now altered so that instead of starting and finishing at A, the route starts at one vertex and finishes at another vertex.

d **i** State the starting vertex and the finishing vertex which minimises the total length of the route. Give a reason for your selections.

ii State the length of your route.

e Explain why, in any network, there is always an even number of vertices of odd degree. ***E***

Solution:

a The route inspection algorithm – description in main text on page 69.

b Odd vertices B, D, F, H

Considering all complete pairings

$BD + FH = 14 + 15 = 29$

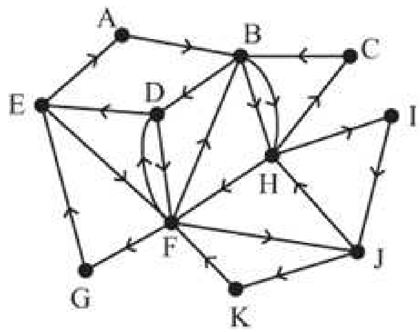
$BF + DH = 10 + 26 = 36$

$BH + DF = 12 + 16 = 28 \leftarrow$ least weight

The shortest route DH is DBH.

Repeat BH and DF.

Adding these arcs to the network gives



A possible route is:

A – B – H – C – B – H – I – J – H – F – J – K – F – B – D – F – D – E – F – G – E – A.

c length of route $= 249 + 28 = 277$

d **i** We will still have to repeat the shortest path between a pair of the odd nodes.
We will choose the pair that requires the shortest path.
The shortest path of the six is BF (10).
We will use D and H as the start and finish nodes.

ii $249 + 10 = 259$

e Each edge, having two ends, contributes two to the sum of valencies for the network.
Therefore the sum $= 2 \times$ number of edges
The sum is even so any odd valencies must occur in pairs.

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Exercise A, Question 1

Question:

The steps involved in starting a car and moving forwards in a straight line are given below.

A Check that car is in neutral.
B Start engine.
C Depress clutch.
D Select first gear.
E Check that it is safe to move off.
F Release the handbrake.
G Raise the clutch and depress the accelerator.

Draw a precedence table for this process.
(There is more than one possible solution.)

Solution:

One possible solution is:

Activity	Depends on
A	–
B	A
C	B
D	C
E	D
F	E
G	F

Another possible solution is:

Activity	Depends on
A	–
B	A
C	–
D	B, C
E	D
F	E
G	F

You can start the engine with the clutch depressed.

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Exercise A, Question 2

Question:

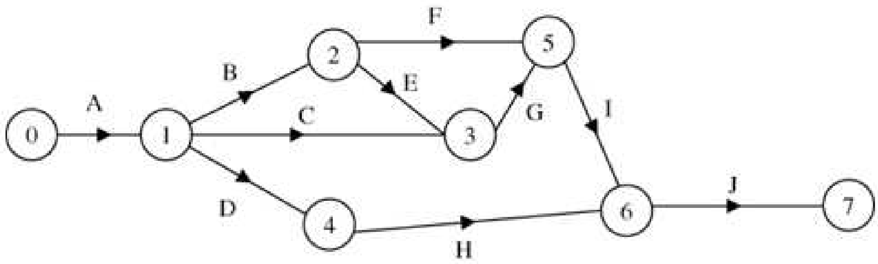
The development of a commercial computer program is divided into activities A to J.
Activity A does not depend on any other activity.
Activities B, C and D all require that Activity A is completed before they can start.
Activities E and F depend on activity B.
Activity G cannot be started until activities C and E have been completed.
Activity H requires the completion of activity D, while activity I requires that both activities F and G are completed first.
Activity J requires the completion of all activities before it may be started.

a Draw a precedence table to represent the development of the computer program.

b Use the precedence table to draw the corresponding activity network.

Solution:

Activity	Depends on
A	–
B	A
C	A
D	A
E	B
F	B
G	C, E
H	D
I	F, G
J	H, I



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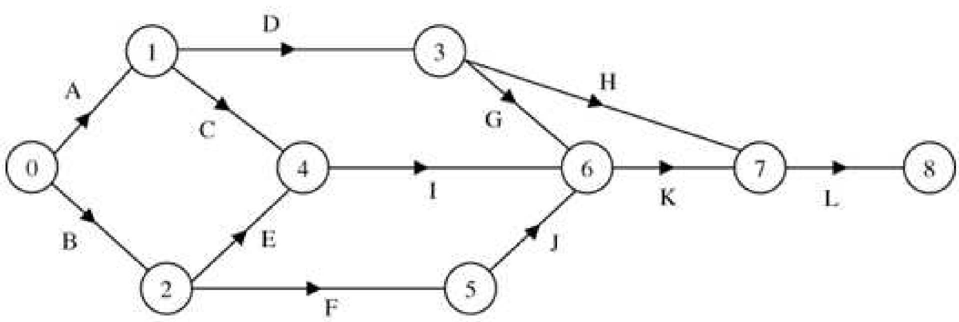
Exercise A, Question 3

Question:

The precedence table for a project is shown below.
Draw the corresponding activity network.

Activity	Depends on
A	—
B	—
C	A
D	A
E	B
F	B
G	D
H	D
I	C, E
J	F
K	G, I, J
L	H, K

Solution:



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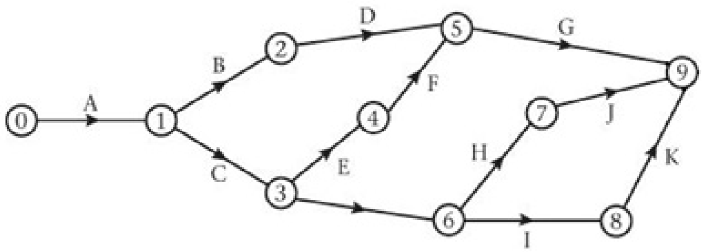
Exercise A, Question 4

Question:

Here is an activity network for a project.



Draw a precedence table to represent the project.

Solution:

Activity	Depends on
A	–
B	A
C	A
D	B
E	C
F	E
G	C
H	D, F
I	G
J	H
K	I
L	J

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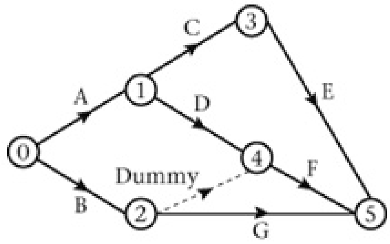
Exercise B, Question 1

Question:

Draw the precedence table for this activity network.



Explain the purpose of the dummy.

Solution:

Activity	Depends on
A	–
B	–
C	A
D	A
E	C
F	B, D
G	B

The dummy shows that activity F depends on activities B and D, whereas activity G only depends on activity B.

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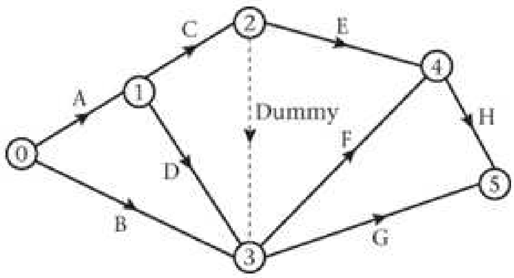
Exercise B, Question 2

Question:

This activity network contains a dummy.



Draw a precedence table for the network.

Solution:

Activity	Depends on
A	–
B	–
C	A
D	A
E	C
F	B, C, D
G	B, C, D
H	E, F

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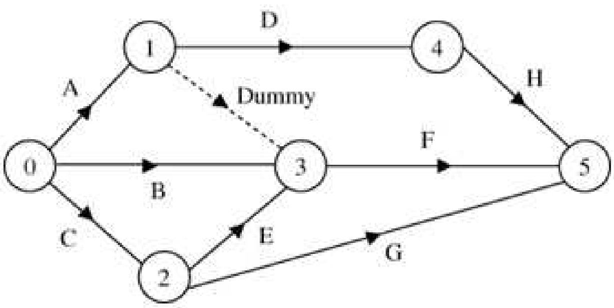
Exercise B, Question 3

Question:

Draw an activity on arc network to represent the precedence table below. Your network should contain exactly one dummy.

Activity	Must be preceded by
A	–
B	–
C	–
D	A
E	C
F	A, B, E
G	C
H	D

Solution:



Solutionbank D1
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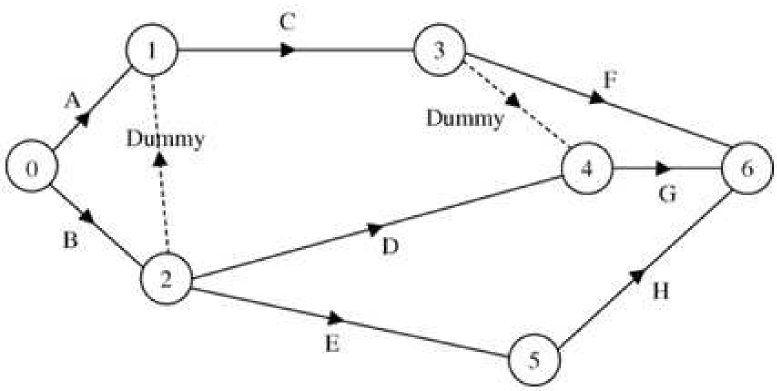
Exercise B, Question 4

Question:

Draw an activity on arc network to represent the precedence table below.
Your network should contain exactly two dummies.

Activity	Depends on
A	—
B	—
C	A, B
D	B
E	B
F	C
G	C, D
H	E

Solution:



Solutionbank D1
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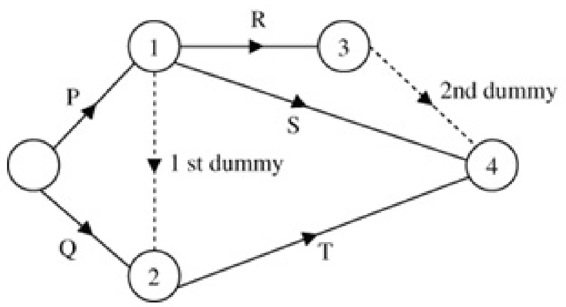
Exercise B, Question 5

Question:

Draw an activity on arc network for this precedence table using exactly two dummies.
Explain the purpose of each dummy.

Activity	Depends on
P	—
Q	—
R	P
S	P
T	P, Q

Solution:



1st dummy ②.
S depends on P only.
T depends on P and Q.

2nd dummy.
So that S and R don't share a start and end event.

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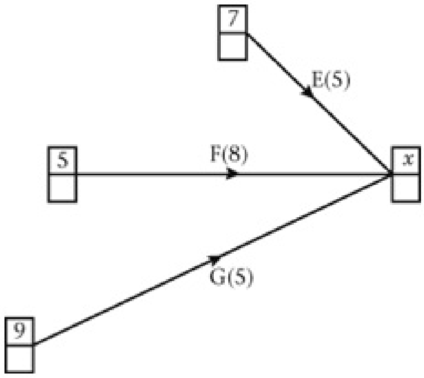
Exercise C, Question 1

Question:

The diagram shows part of an activity network.
Calculate the value of x.



Solution:

x is the largest of $7+5=12$, $5+8=13$ and $9+5=14$.
$x=14$

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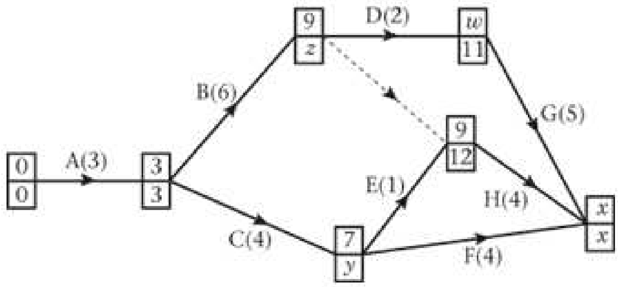
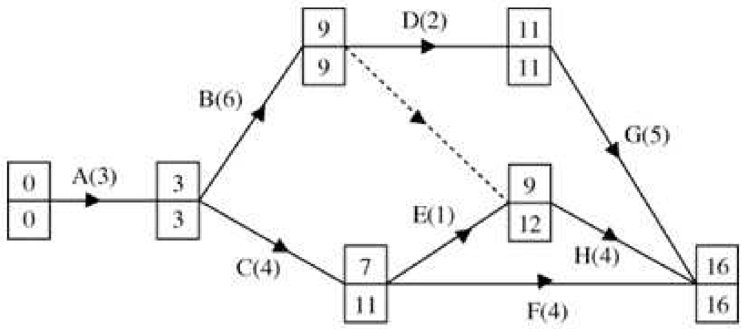
Exercise C, Question 2

Question:

The activity network for a project is given opposite.
The time in hours needed to complete each activity is shown in brackets.
Early and late times are shown at each vertex.
Calculate the values of w, x, y and z.



Solution:



$w = 11, x = 16, y = 11, z = 9$

w and x are found using a forward scan.
y and z are found using a backward scan.

Solutionbank D1

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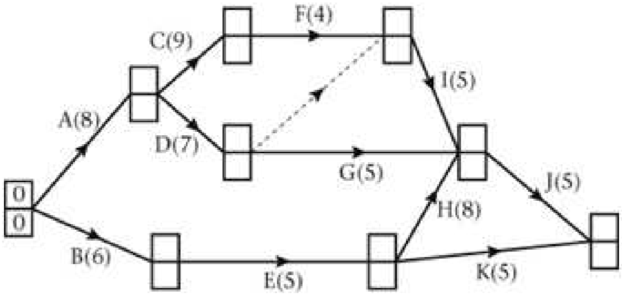
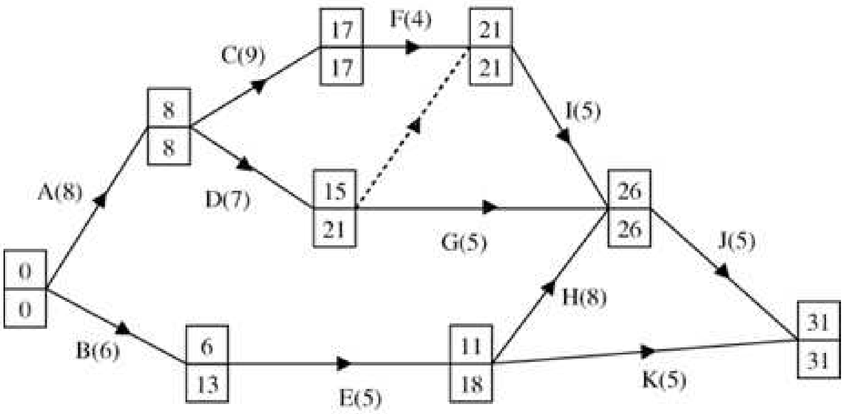
Exercise C, Question 3

Question:

The activity network for a project is given opposite.
The time in days needed to complete each activity is shown in brackets.
Calculate the early and late times at each vertex.



Solution:



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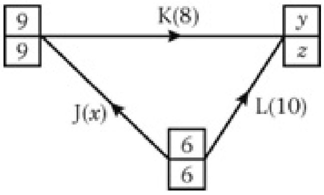
Exercise D, Question 1

Question:

Part of an activity network is shown opposite including the early and late event times given in hours.
Activities J and K are critical.
Find the values of x, y and z.



Solution:

$x = 3, y = 17, z = 17$

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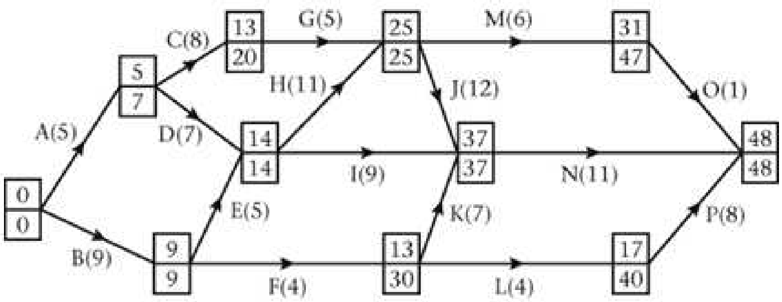
Exercise D, Question 2

Question:

The diagram shows an activity network with early and late event times, in hours, shown at the vertices.

a Identify the critical activities.

b Name an activity that links two critical events but is not critical.



Solution:

a The critical activities are B, E, H, J and N

b I

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Exercise D, Question 3

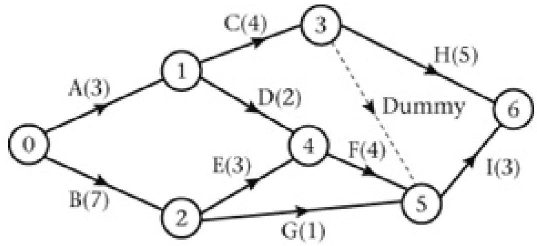
Question:

The activity network for a project is shown below. Activity times are given in days and are shown in brackets.

a Copy and complete the activity network to show the early and late event times.

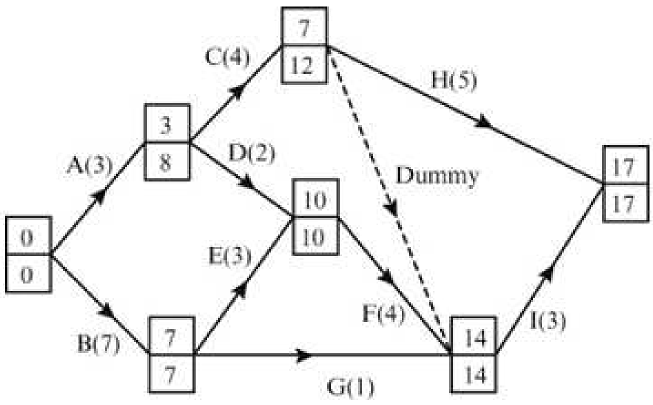
b Is G a critical activity?
Explain your answer.

c Describe the critical path.



Solution:

a



b No. $7+1 \neq 14$

c The critical path is B–E–F–I.

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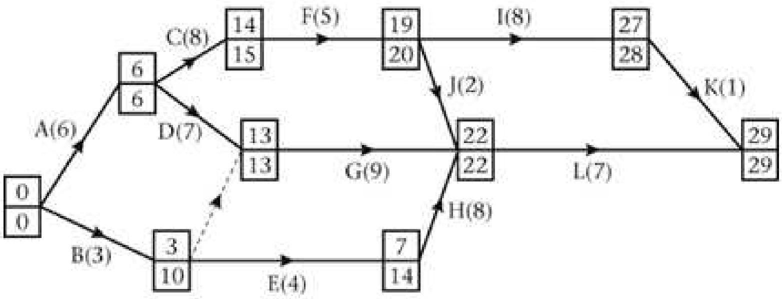
Exercise E, Question 1

Question:

Determine the total float of each activity in this activity network.



Solution:

Activity	Total float
A	0
B	10 – 3 – 0 = 7
C	15 – 8 – 6 = 1
D	0
E	14 – 4 – 3 = 7
F	20 – 5 – 14 = 1
G	0
H	22 – 8 – 7 = 7
I	28 – 8 – 19 = 1
J	22 – 2 – 19 = 1
K	29 – 1 – 27 = 1
L	0

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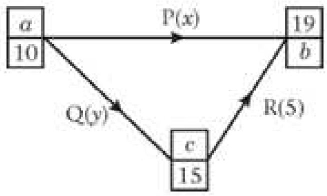
Exercise E, Question 2

Question:

The diagram shows part of an activity network with activity times measured in hours.

P is a critical activity.
Q has a total float of 3 hours.



a Work out the values of a, b, x and y.

b What is the minimum possible value of c?

c What is the maximum possible value of the total float of R?

Solution:

a $a = 10$ $\quad b = 19 \quad$ $x = 19 - 10 = 9$

Total float $= 3 = 15 - y - a$

$y = 15 - 3 - 10$

$y = 2$

b Minimum value of $c = 10 + 2 = 12$

c Minimum value of total float of R $= 19 - 5 - 12$

$= 2$

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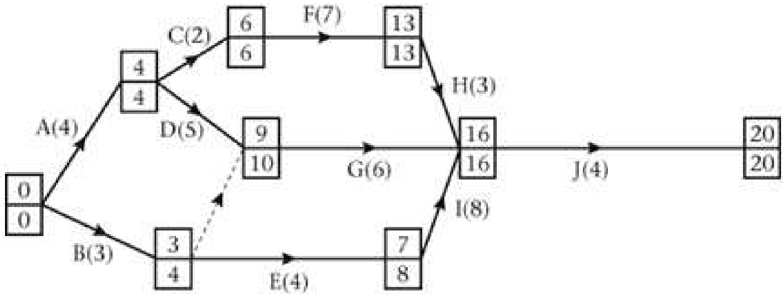
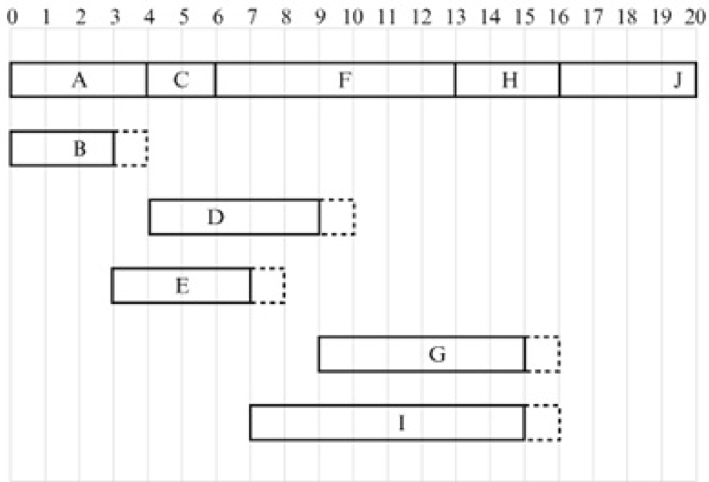
Exercise F, Question 1

Question:

The diagram shows an activity network for a project. Early and late event times are shown in days at the nodes. Draw a Gantt chart to represent the project.



Solution:



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Exercise F, Question 2

Question:

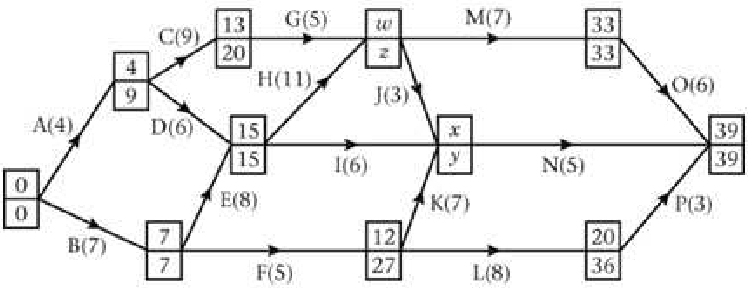
An activity network for a project is shown below.

a Calculate the values of w, x, y and z.

b List the critical activities.

c Calculate the total float for activities G and N.

d Draw a Gantt chart to represent the project.



Solution:

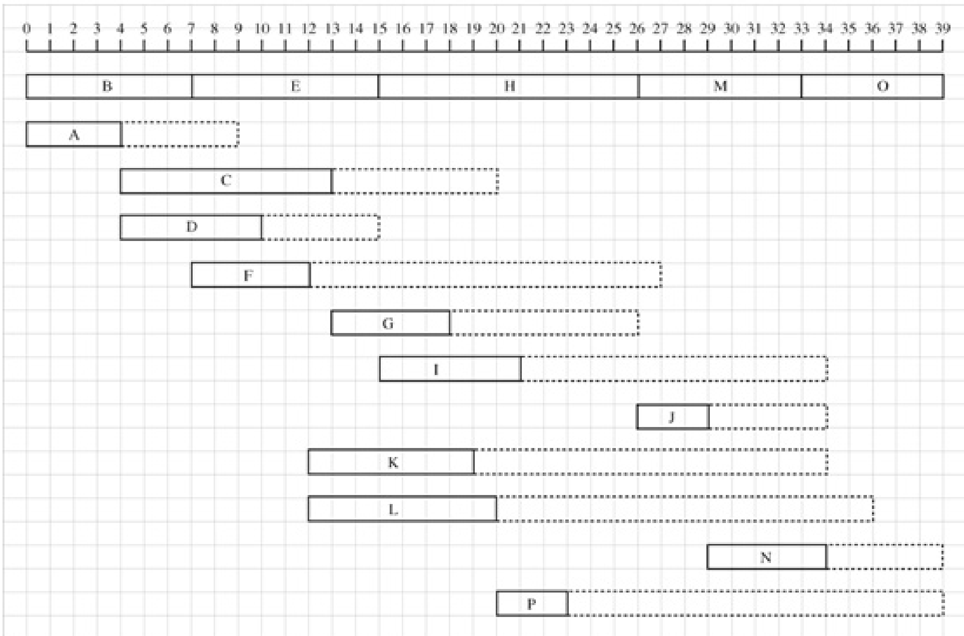
a $w = 26$ $x = 29$ $y = 34$ $z = 26$

b Critical activities: B, E, H, M, O

c Total float for E = $26 - 5 - 13 = 8$

Total float for N = $39 - 5 - 29 = 5$

d



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Exercise G, Question 1

Question:

Refer to the Gantt chart shown in Example 12 for this question.

a Which activities *must* be happening at midday on day 8?

b Which activities *must* be happening at midday on day 21?

c Which activities *may* be happening at midday on day 22?

Solution:

a A, E

b G, H

c F, H

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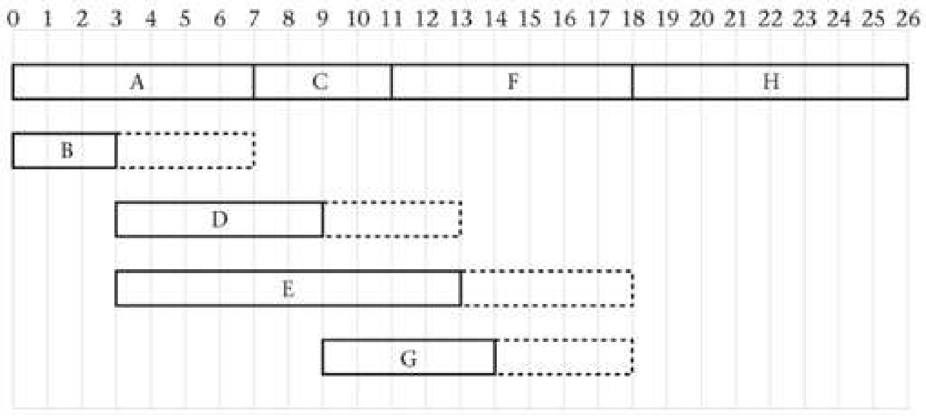
Exercise G, Question 2

Question:

The Gantt chart below represents an engineering project. An engineer decides to carry out some spot checks on the progress of the project.

a Which activities *must* be happening at 12 noon on day 8?

b Which activities *may* be happening at 12 noon on day 15?



Solution:

a C, D

b E, G

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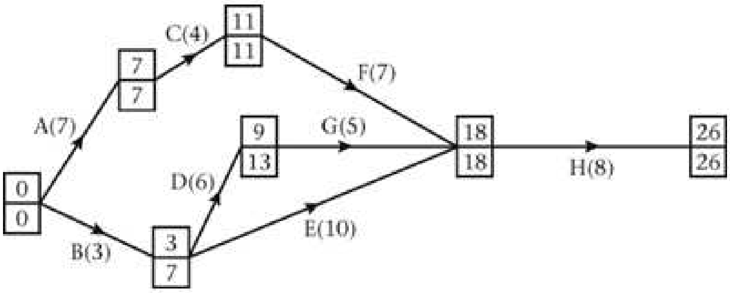
Exercise G, Question 3

Question:

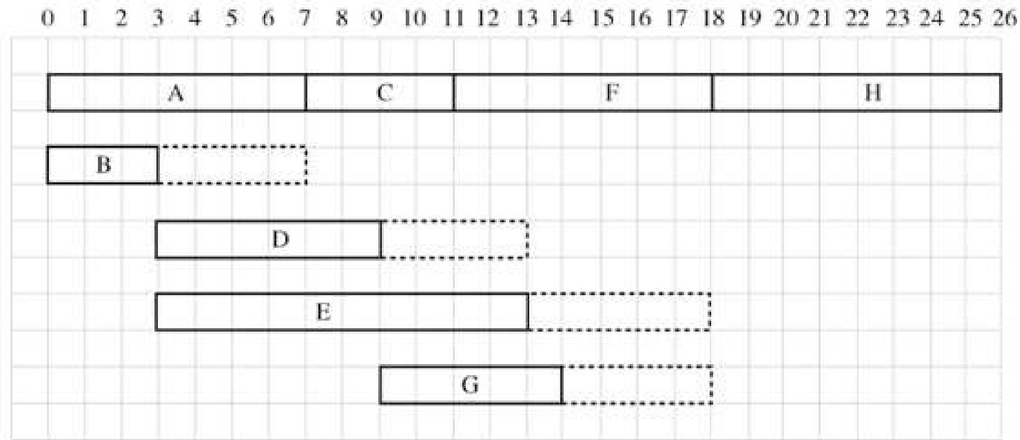
Draw a Gantt chart to represent the activity network below.
Use your chart to determine:

a which activities *may* be happening at midday on day 5,

b which activities *must* be happening at midday on day 7.



Solution:



a B, D and E may be happening at midday on day 5,

b Only A must be happening at midday on day 7.

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Exercise H, Question 1

Question:

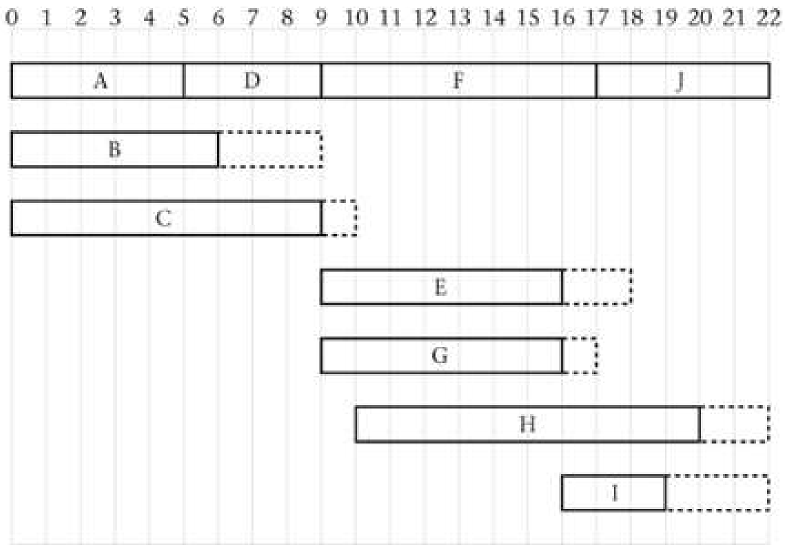
The cascade chart below represents a project with a critical time of 22 hours.

a Given that the total duration of all of the activities is 64 hours calculate a lower bound for the number of workers needed to complete the project in the minimum time.

b An unforeseen problem means that Activity B cannot be started until 2 hours into the project. Does this mean that the time for the whole project is delayed?

c Which activity *must* be happening 17 hours into the project?

d Complete a scheduling diagram to complete the project in 22 hours. State the number of workers required.



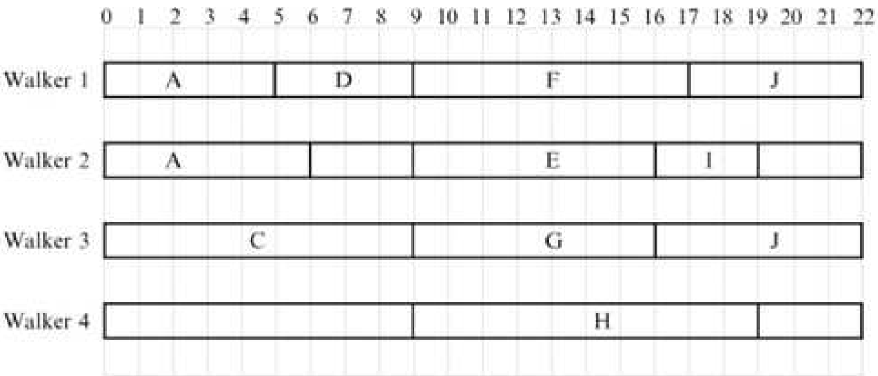
Solution:

a $\frac{64}{22} = 2.9\ldots$ so, lower bound $= 3$

b No. 2 hours is less than the total float for activity B (3 hours).

c Activity H

d



4 workers are needed to complete the project in 22 hours.

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Exercise H, Question 2

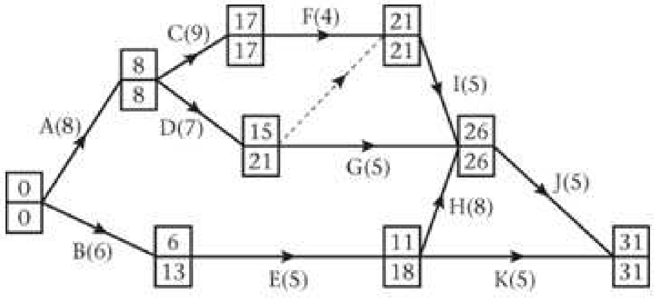
Question:

The activity network used in Example 14 is shown again here.

a Draw a Gantt chart to represent the project.

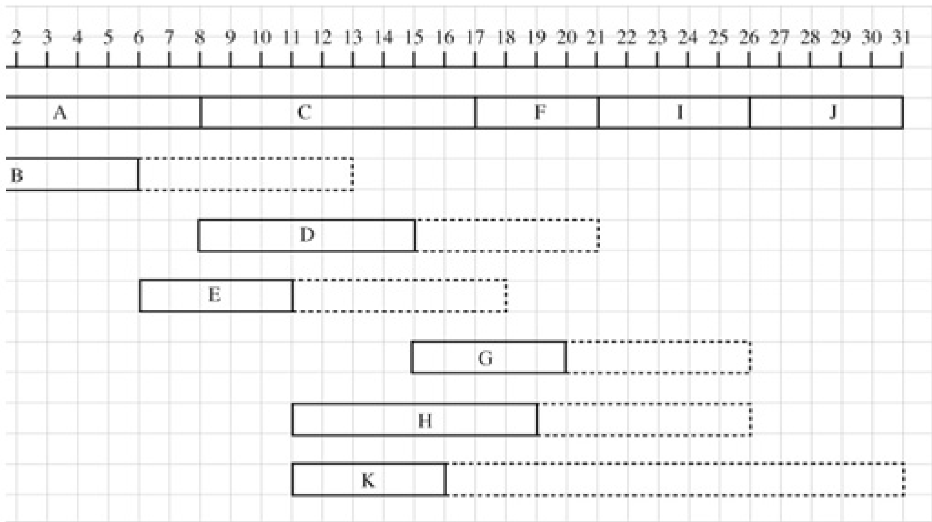
b Schedule the project to be completed by the minimum number of workers in the critical time.

State the number of workers required.

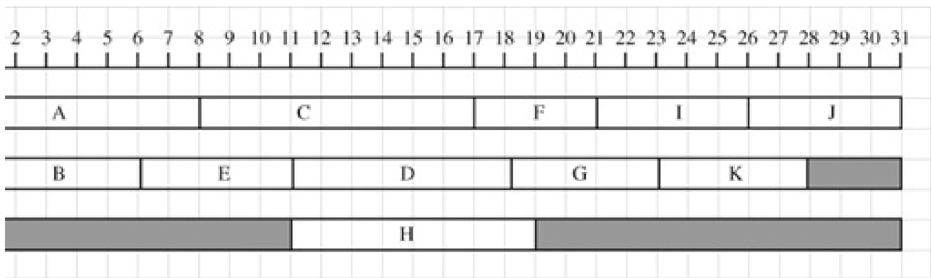


Solution:

a



b



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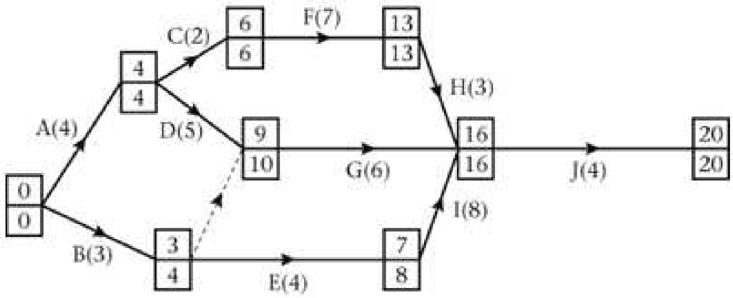
Exercise H, Question 3

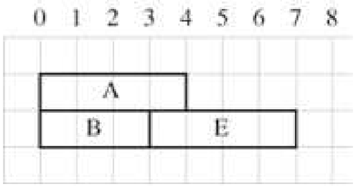
Question:

Construct a scheduling diagram based on the activity network below, given that only two workers are available.

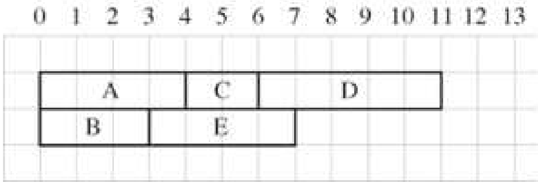
Find the new minimum time for completion of the project.



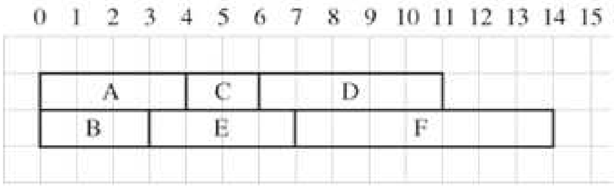
Solution:



When worker 2 completes activity B, only activity E may be started.

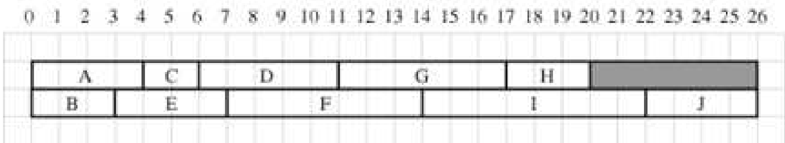


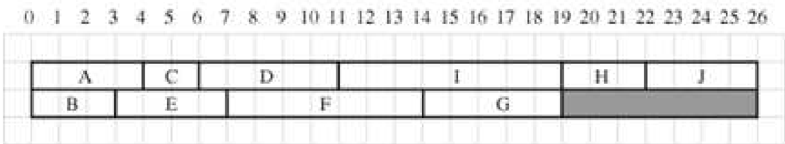
When worker 1 completes activity A, the next activity to start is either C or D. Activity C is chosen because it has the lower value for its latest finish time.



At this stage, worker 1 has a choice between activity G and activity I. Activity H may not be started until activity F has been completed.

There are two possible ways to complete the schedule so that the project is completed in the minimum possible time.





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Exercise I, Question 1

Question:

The precedence table for activities involved in producing a computer game is shown opposite. An activity on arc network is to be drawn to model this production process.

a Explain why it is necessary to use at least two dummies when drawing the activity network.

b Draw the activity network using exactly two dummies.

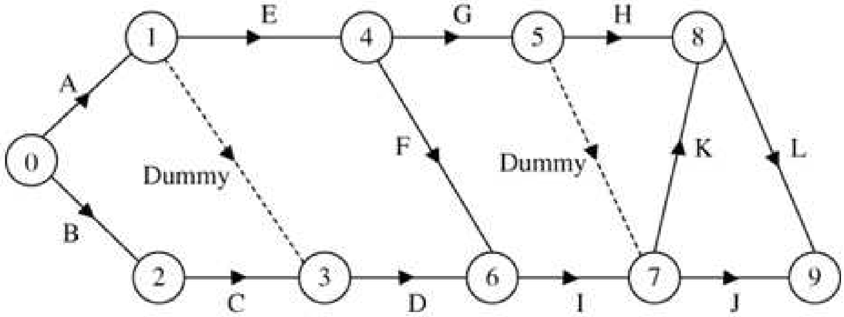
Activity	Must be preceded by
A	—
B	—
C	B
D	A, C
E	A
F	E
G	E
H	G
I	D, F
J	G, I
K	G, I
L	H, K

Solution:

a Activity D depends on activities A and C, whereas activity E depends only on activity A. This shows that a dummy is required.

Activity J depends on activities G and I, whereas activity H depends only on activity G. This shows that a second dummy is required.

b



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Exercise I, Question 2

Question:

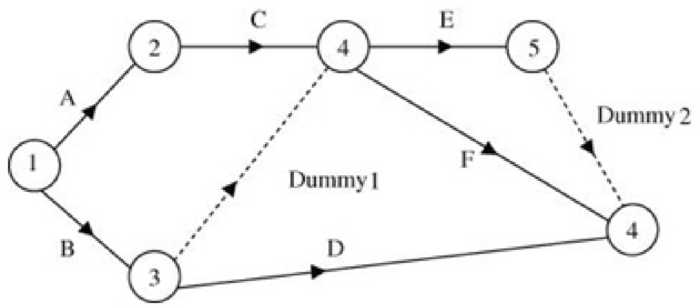
a Draw the activity network described in this precedence table, using activity on arc and exactly two dummies.

Activity	Immediately preceding activities
A	–
B	–
C	A
D	B
E	B, C
F	B, C

b Explain why each of the two dummies is necessary.

Solution:

a



b Dummy1 is needed to show *dependency*.
E and F depend on C and B, but D depends on B only.

Dummy2 is needed so that each activity can be *uniquely* represented in terms of its event.

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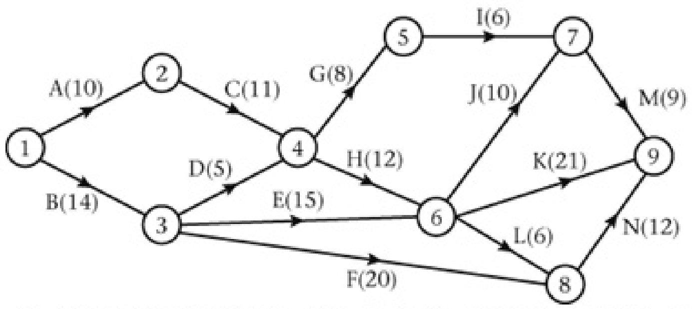
Exercise I, Question 3

Question:



An engineering project is modelled by the activity network shown above. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. Each activity requires one worker. The project is to be completed in the shortest time.

a Calculate the early time and late time for each event.

b State the critical activities.

c Find the total float on activities D and F. You must show your working.

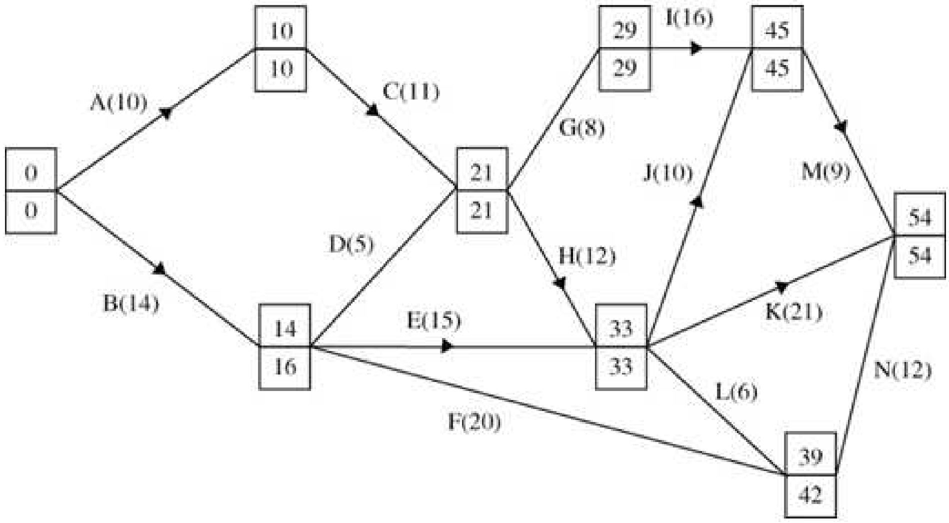
d Draw a cascade (Gantt) chart for this project.

The chief engineer visits the project on day 15 and day 25 to check the progress of the work. Given that the project is on schedule,

e which activities *must* be happening on each of these two days? ***E***

Solution:

a

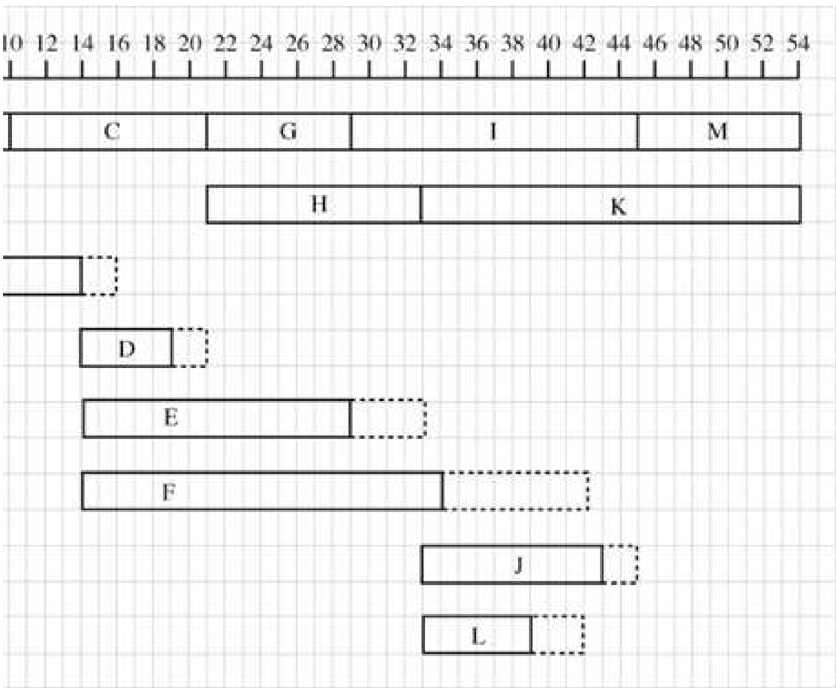


b There are *two* critical paths:
A–C–G–I–M and A–C–H–K

The critical activities are A, C, G, H, I, K

c Total float on D is $21 - 5 - 14 = 2$
Total float on F is $42 - 20 - 14 = 8$

d



e Day 15 : C
Day 25: G, H, E, F

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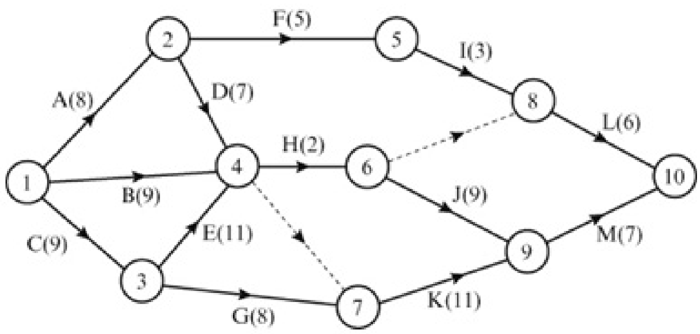
Exercise I, Question 4

Question:



A project is modelled by the activity network shown above. The activities are represented by the arcs. The number in brackets on each arc gives the time, in days, to complete the activity. The numbers in circles are the event numbers. Each activity requires one worker.

a Explain the purpose of the dotted line from event 6 to event 8.

b Calculate the early time and late time for each event.

c Calculate the total float on activities D, E and F.

d Determine the critical activities.

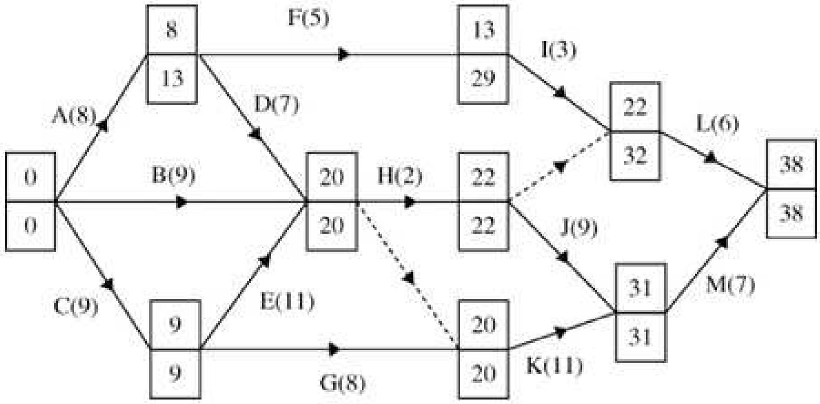
e Given that the sum of all the times of the activities is 95 hours, calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

f Given that workers may not share an activity, schedule the activities so that the process is completed in the shortest time using the minimum number of workers. ***E***

Solution:

a J depends on H alone, but L depends on H and I.

b

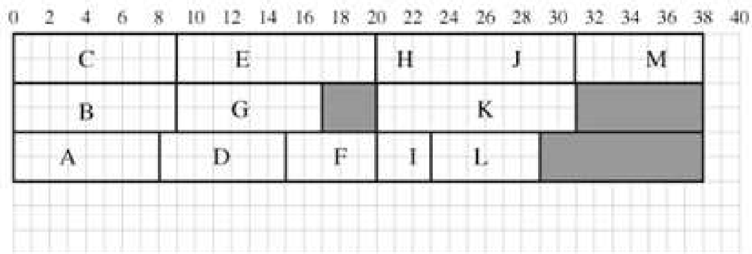


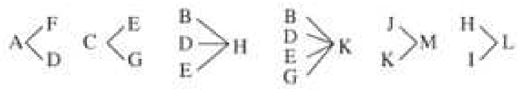
c Total float on D = 20 − 7 − 8 = 5
Total float on E = 20 − 11 − 9 = 0
Total float on F = 29 − 5 − 8 = 16

d

e $\frac{95}{38} = 2.5$ so 3 workers

f For example





Solutionbank D1

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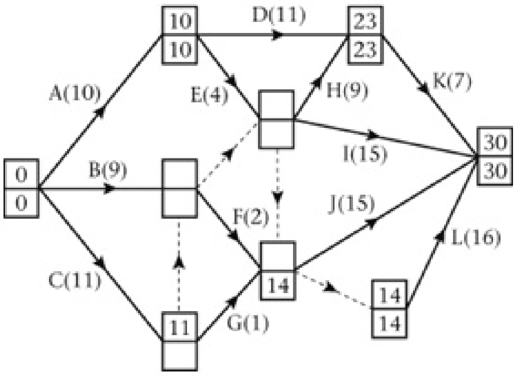
Exercise I, Question 5

Question:

The network shows the activities that need to be undertaken to complete a project. Each activity is represented by an arc. The number in brackets is the duration of the activity in days. The early and late event times are to be shown at each vertex and some have been completed for you.



a Calculate the missing early time and late times.

b List the two critical paths for this network.

c Explain what is meant by a critical path.

The sum of all the activity times is 110 days and each activity requires just one worker. The project must be completed in the minimum time.

d Calculate a lower bound for the number of workers needed to complete the project in the minimum time. You must show your working.

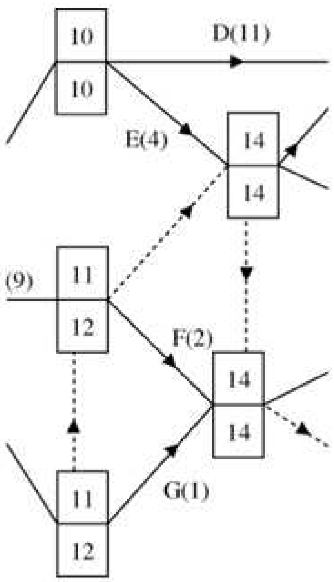
e List the activities that must be happening on day 20.

f Comment on your answer to part **e** with regard to the lower bound you found in part **d**.

g Schedule the activities, using the minimum number of workers, so that the project is completed in 30 days. ***E***

Solution:

a



b The critical paths are: A–E–H–K and A–E–L.

c A critical path is a continuous path from the source node to the sink node such that a delay in any activity results in a corresponding delay in the whole project.

d

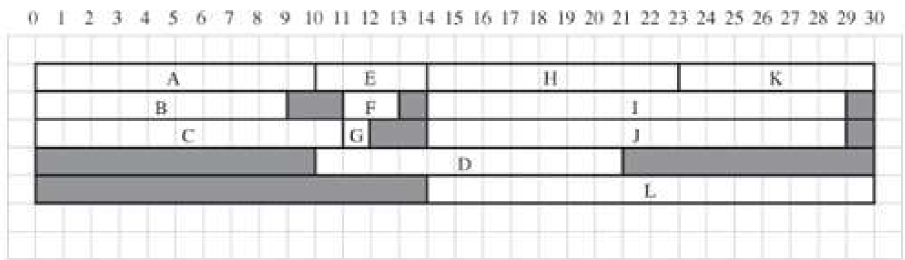
$$\frac{\text{Sum of all of the activity times}}{\text{critical time of the project}} = \frac{110}{30}$$

Lower bound for number of workers is 4.

e D, H, I, J, L

f The answers to part **e** show that 5 workers are needed on day 20 in order to complete the project in the minimum time.

g



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Exercise A, Question 1

Question:

A chocolate manufacturer is producing two hand-made assortments, gold and silver, to commemorate 50 years in business.

It will take 30 minutes to make all the chocolates for one box of gold assortment and 20 minutes to make the chocolates for one box of silver assortment.

It will take 12 minutes to wrap and pack the chocolates in one box of gold assortment and 15 minutes for one box of silver assortment.

The manufacturer needs to make at least twice as many silver as gold assortments.

The gold assortment will be sold at a profit of 80p, and the silver at a profit of 60p.

There are 300 hours available to make the chocolates and 200 hours to wrap them. The profit is to be maximised.

Letting the number of boxes of gold assortment be x and the number of boxes of silver assortment be y, formulate this as a linear programming problem.

Solution:

Number of boxes of gold assortment $= x$
Number of boxes of silver assortment $= y$
Objective: maximise $P = 80x + 60y$

Constraints

- time to make chocolate, $30x + 20y \le 300 \times 60$
  which simplifies to $3x + 2y \le 1800$
- time to wrap and pack $12x + 15y \le 200 \times 60$
  which simplifies to $4x + 5y \le 4000$

All units now in minutes.

- 'At *least* twice as many silver as gold' $2x \le y$

- non-negativity $x, y \ge 0$

In summary: maximise $P = 80x + 60y$
subject to

$$3x + 2y \le 1800$$
$$4x + 5y \le 4000$$
$$2x \le y$$
$$x, y \ge 0$$

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Exercise A, Question 2

Question:

A floral display is required for the opening of a new building. The display must be at least 30 m long and is to be made up of two types of planted displays, type A and type B.

Type A is 1 m in length and costs £6
Type B is 1.5 m in length and costs £10

The client wants at least twice as many type A as type B, and at least 6 of type B.
The cost is to be minimised.

Letting x be the number of type A used and y be the number of type B used, formulate this as a linear programming problem.

Solution:

number of type A $= x$
number of type B $= y$
Objective: minimise $C = 6x + 10y$

Constraints

- Display must be at least 30 m long $x + 1.5y \geq 30$
  which simplifies to $2x + 3y \geq 60$
- 'At least twice as many x as y' $2y \leq x$
- At *least* six type B $y \geq 6$
- non-negativity $x, y \geq 0$

In summary: minimise $C = 6x + 10y$
subject to:

$$2x + 3y \geq 60$$
$$2y \leq x$$
$$y \geq 6$$
$$x, y \geq 0$$

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Exercise A, Question 3

Question:

A toy company makes two types of board game, Cludopoly and Trivscrab. As well as the board each game requires playing pieces and cards.

The company uses two machines, one to produce the pieces and one to produce the cards. Both machines can only be operated for up to ten hours per day.

The first machine takes 5 minutes to produce a set of pieces for Cludopoly and 8 minutes to produce a set of pieces for Trivscrab.

The second machine takes 8 minutes to produce a set of cards for Cludopoly and 4 minutes to produce a set of cards for Trivscrab.

The company knows it will sell at most three times as many games of Cludopoly as Trivscrab.

The profit made on each game of Cludopoly is £1.50 and £2.50 on each game of Trivscrab.
The company wishes to maximise its daily profit.
Let x be the number of games of Cludopoly and y the number of games of Trivscrab.
Formulate this problem as a linear programming problem.

Solution:

number of games of Cludopoly $= x$
number of games of Trivscrab $= y$
Objective: maximise $P = 1.5x + 2.5y$

Constraints

- First machine. $5x + 8y \leq 10 \times 60$
  which simplifies to $5x + 8y \leq 600$
- Second machine: $8x + 4y \leq 10 \times 60$
  which simplifies to $2x + y \leq 150$
- At *most* 3 times as many x as y $3y \geq x$
- non-negativity $x, y \geq 0$

All units now in minutes.

In summary: maximise $P = 1.5x + 2.5y$
subject to:

$$5x + 8y \leq 600$$
$$2x + y \leq 150$$
$$3y \geq x$$
$$x, y \geq 0$$

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Exercise A, Question 4

Question:

A librarian needs to purchase bookcases for a new library. She has a budget of £3000 and 240 m^2 of available floor space. There are two types of bookcase, type 1 and type 2, that she is permitted to buy.
Type 1 costs £150, needs 15 m^2 of floor space and has 40 m of shelving.
Type 2 costs £250, needs 12 m^2 of floor space and has 50 m of shelving.
She must buy at least 8 type 1 bookcases and wants at most $\frac{1}{3}$ of all the bookcases to be type 2.
She wishes to maximise the total amount of shelving.
Letting x and y be the number of type 1 and type 2 bookcases bought respectively, formulate this as a linear programming problem.

Solution:

Number of type 1 bookcases $= x$
Number of type 2 bookcases $= y$

Objective: maximise $S = 40x + 60y$

Constraints

- budget $150x + 250y \le 3000$
  which simplifies to $3x + 5y \le 60$
- floor space $15x + 12y \le 240$
  which simplifies to $5x + 4y \le 80$
- 'At most $\frac{1}{3}$ of all bookcases to be type 2' $y \le \frac{1}{3}(x+y)$
  which simplifies to $2y \le x$
- At least 8 type 1 $x \ge 8$
- non-negativity $x, y \ge 0$

In summary: maximise $S = 40x + 60y$
subject to:

$$3x + 5y \le 60$$
$$5x + 4y \le 80$$
$$2y \le x$$
$$x \ge 8$$
$$x, y \ge 0$$

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Exercise A, Question 5

Question:

A garden supplies company produces two different plant feeds, one for indoor plants and one for outdoor plants.

In addition to other ingredients, the plant feeds are made by combining three different natural ingredients A, B and C.

Each kilogram of indoor feed requires 10 g of A, 20 g of B and 20 g of C.
Each kilogram of outdoor feed requires 20 g of A, 10 g of B and 20 g of C.

The company has 5 kg of A, 5 kg of B and 6 kg of C available each week to use to make these feeds.

The company will sell at most three times as much outdoor as indoor feed, and will sell at least 50 kg of indoor feed.

The profit made on each kilogram of indoor and outdoor feed is £7 and £6 respectively. The company wishes to maximise its weekly profit.
Formulate this as a linear programming problem, defining your decision variables.

Solution:

Let $x =$ number of kg of indoor feed and $y =$ number of kg of outdoor feed
Objective: Maximise $P = 7x + 6y$
Constraints

- Amount of A $10x + 20y \le 5 \times 1000$
  which simplifies to $x + 2y \le 500$
- Amount of B $20x + 10y \le 5 \times 1000$
  which simplifies to $2x + y \le 500$
- Amount of C $20x + 20y \le 6 \times 100$
  which simplifies to $x + y \le 300$

All units now in grams.

- At *most* 3 times as much y as x $y \le 3x$
- At least 50 kg of x $x \ge 50$
- non-negativity $y \ge 0$ ($x \ge 0$ is unnecessary because of the previous constraint)

In summary:
maximise $P = 7x + 6y$
subject to

$$\begin{aligned} x + 2y &\le 500 \\ 2x + y &\le 500 \\ x + y &\le 300 \\ y &\le 3x \\ x &\ge 50 \\ y &\ge 0 \end{aligned}$$

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Exercise A, Question 6

Question:

Sam makes three types of fruit smoothies, A, B and C. As well as other ingredients all three smoothies contain oranges, raspberries, kiwi fruit and apples, but in different proportions. Sam has 50 oranges, 1000 raspberries, 100 kiwi fruit and 60 apples. The table below shows the number of these 4 fruits used to make each smoothie and the profit made per smoothie. Sam wishes to maximise the profit.

Smoothie	Oranges	Raspberries	Kiwi fruit	Apples	Profit
A	1	10	2	2	60p
B	$\frac{1}{2}$	40	3	$\frac{1}{2}$	65p
C	2	15	1	2	55p
Total available	50	1000	100	60	

Letting x be the number of A smoothies, y the number of B smoothies and z the number of C smoothies, formulate this as a linear programming problem.

Solution:

number of A smoothies $= x$
number of B smoothies $= y$
number of C smoothies $= z$

Objective maximise $P = 60x + 65y + 55z$
Constraints

- oranges $x + \frac{1}{2}y + 2z \le 50$
  which simplifies to $2x + y + 4z \le 100$
- raspberries $10x + 40y + 15z \le 1000$
  which simplifies to $2x + 8y + 3z \le 200$
- kiwi fruit $2x + 3y + z \le 100$
- apples $2x + \frac{1}{2}y + 2z \le 60$
  which simplifies to $4x + y + 4z \le 120$
- non-negativity $x, y, z \ge 0$

In summary: maximise $P = 60x + 65y + 55z$
subject to:

$$\begin{aligned} 2x + y + 4z &\le 100 \\ 2x + 8y + 3z &\le 200 \\ 2x + 3y + z &\le 100 \\ 4x + y + 4z &\le 120 \\ x, y, z &\ge 0 \end{aligned}$$

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Exercise A, Question 7

Question:

A dairy manufacturer has two factories R and S. Each factory can process milk and yoghurt.
Factory R can process 1000 litres of milk and 200 litres of yoghurt per hour.
Factory S can process 800 litres of milk and 300 litres of yoghurt per hour.
It costs £300 per hour to operate factory R and £400 per hour to operate factory S. In order to safeguard jobs it has been agreed that each factory will operate for at least $\frac{1}{3}$ of the total, combined, operating time.
The manufacturer needs to process 20 000 litres of milk and 6000 litres of yoghurt. He wishes to distribute this between the 2 factories in such a way as to minimise operating costs.
Formulate this as a linear programming problem in x and y, defining your decision variables.

Solution:

Let number of hours of work for factory R $= x$
Let number of hours of work for factory S $= y$

Objective: minimise $C = 300x + 400y$

Constraints

- milk $1000x + 800y \geq 20\,000$
  which simplifies to $5x + 4y \geq 100$
- yoghurt $200x + 300y \geq 6000$
  which simplifies to $2x + 3y \geq 60$
- At *least* $\frac{1}{3}$ of total time for R $x \geq \frac{1}{3}(x + y)$
  which simplifies to $2x \geq y$
- At *least* $\frac{1}{3}$ of total time for S $y \geq \frac{1}{3}(x + y)$
  which simplifies to $2y \geq x$
- non-negativity $x, y \geq 0$

In summary minimise $C = 300x + 400y$
subject to:

$$\begin{aligned} 5x + 4y &\geq 100 \\ 2x + 3y &\geq 60 \\ 2x &\geq y \\ 2y &\geq x \\ x, y &\geq 0 \end{aligned}$$

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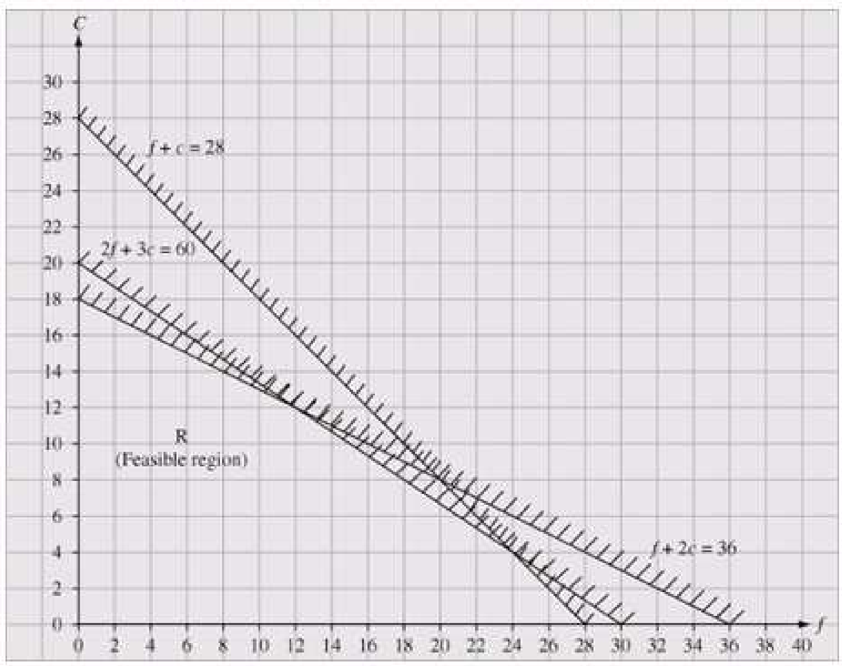
Exercise B, Question 1

Question:

Illustrate the inequalities found in Example 1 (pages 114–115) on graph paper.
Label the feasible region, R.

Solution:



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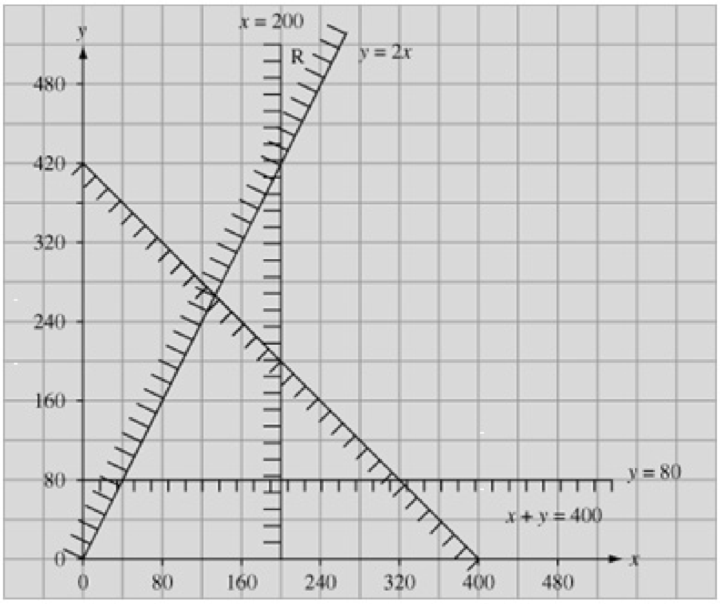
Exercise B, Question 2

Question:

Repeat Question 1 using the inequalities found in Example 2 (pages 116–117).

Solution:



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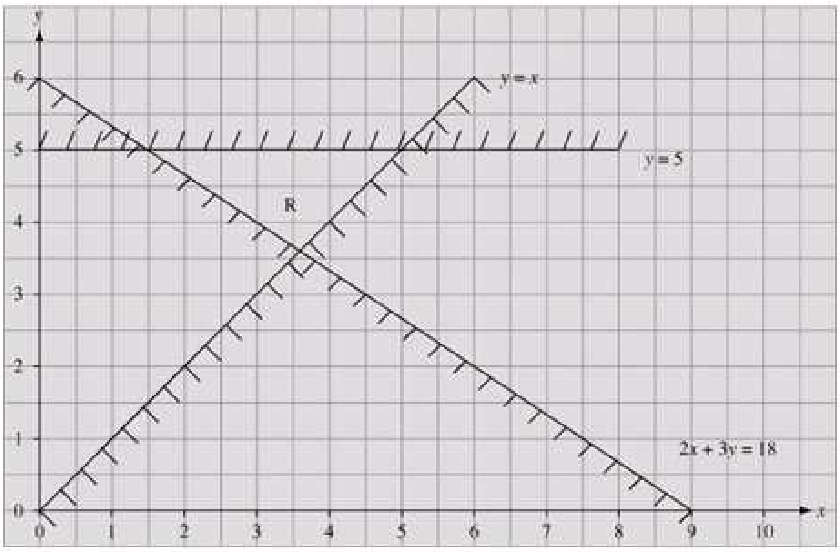
Exercise B, Question 3

Question:

Represent the following inequalities graphically. Label the region bounded by these inequalities R.

$$2x + 3y > 18$$
$$y > x$$
$$y \leq 5$$
$$x, y \geq 0$$

Solution:



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Exercise B, Question 4

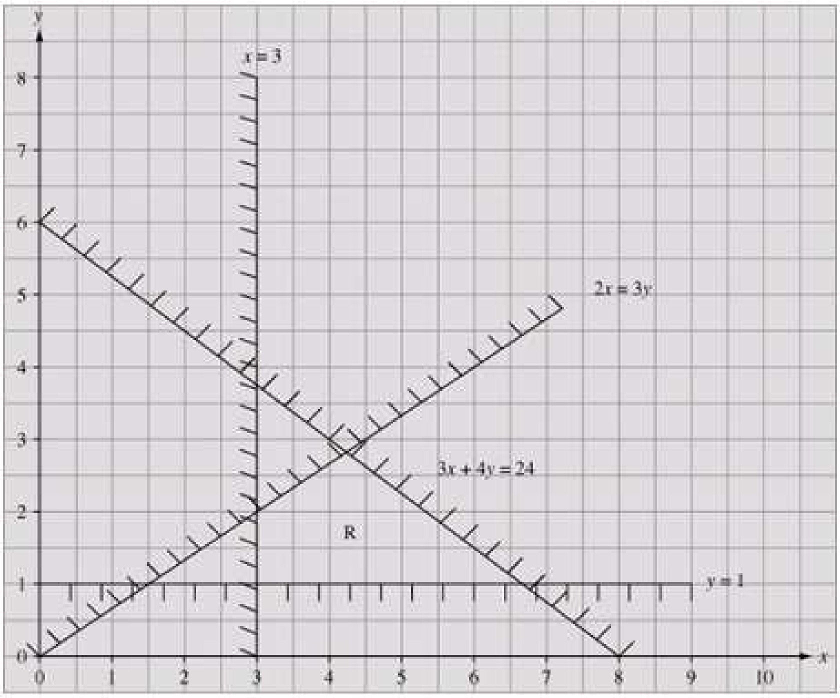
Question:

The following inequalities were found when solving a linear programming problem.

$$2x \le 3y$$
$$3x + 4y \le 24$$
$$x \ge 3$$
$$y \ge 1$$

Represent these inequalities on a graph.
Indicate the feasible region by labelling it R.

Solution:



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Exercise B, Question 5

Question:

Region R is bounded by the following inequalities

$x + y \le 20$

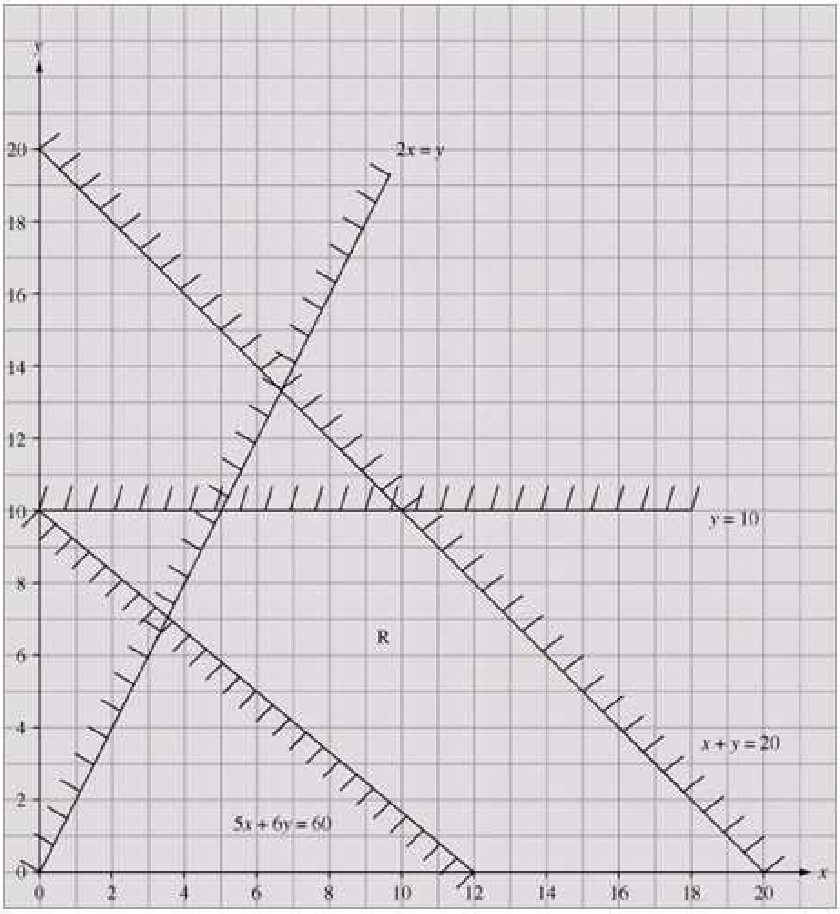
$5x + 6y \ge 60$

$2x \ge y$

$y \le 10$

By drawing suitable straight lines, draw a graph to show region R.

Solution:



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Exercise B, Question 6

Question:

Indicate on a graph the region, R, for which

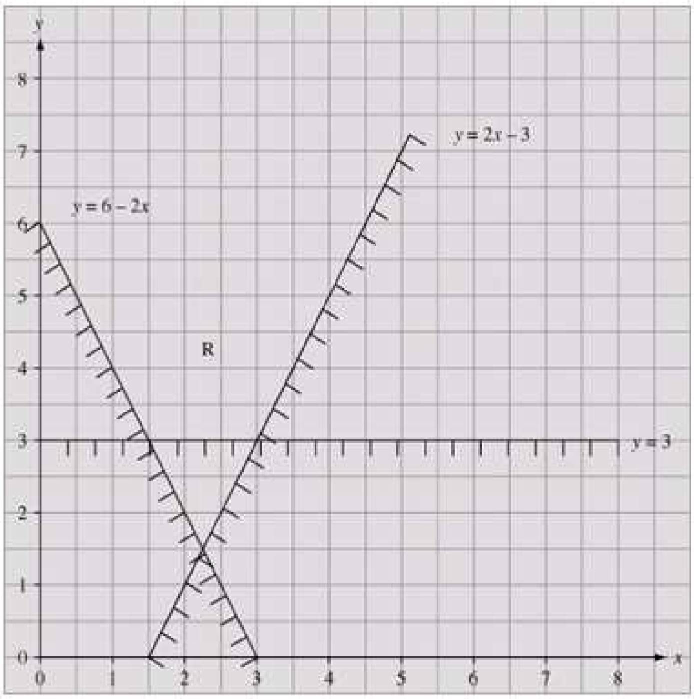
$2x-3<y$

$y>3$

$y>6-2x$

$x \geq 0$

Solution:



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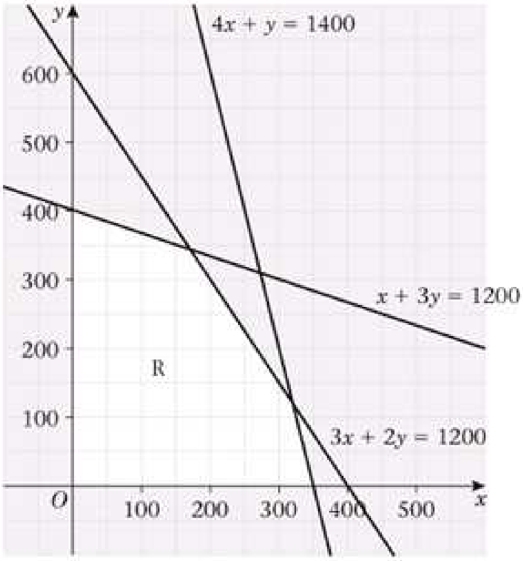
Exercise C, Question 1

Question:



The diagram shows a feasible region, R. Find the optimal point and the optimal value, using

a the objective line method, with the objective 'maximise $M = 2x + y$',

b the objective line method, with the objective 'maximise $N = x + 4y$',

c the vertex testing method, with the objective 'maximise $P = x + y$',

d the vertex testing method, with the objective 'maximise $Q = 6x + y$'.

Solution:

a Need intersection of $4x + y = 1400$ and $3x + 2y = 1200$

Objective line passes through (200, 0) and (0, 400).

(320, 120) $m = 760$

b (0, 400) $N = 1600$

Objective line passes through (400,0) and (100,0).

c Need intersection of $x + 3y = 1200$ and $3x + 2y = 1200$

Objective line passes through (200,0) and (0, 200).

$(171\frac{3}{7}, 342\frac{6}{7})$ $P = 514\frac{2}{7}$

d (350, 0) $Q = 2100$

Objective line passes through (100, 0) and (0, 600).

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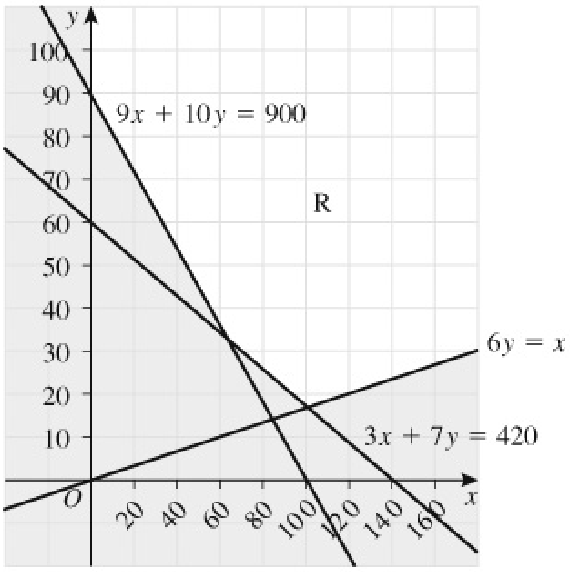
Exercise C, Question 2

Question:



The diagram shows a feasible region, R.

Find the optimal point and the optimal value, using

a the vertex testing method, with the objective 'minimise $E = 2x + y$',

b the vertex testing method, with the objective 'minimise $F = x + 4y$',

c the objective line method, with the objective 'minimise $G = 3x + 4y$',

d the objective line method, with the objective 'minimise $H = x + 6y$',

Solution:

a $(0, 90)$ $E = 90$

b Need intersection of $6y = x$ and $3x + 7y = 420$

$(100.8, 16.8)$ $F = 168$

c Need intersection of $9x + 10y = 900$

$3x + 7y = 420$

$\left(63\frac{7}{11}, 32\frac{8}{11}\right)$ $G = 321\frac{9}{11}$

Objective line passes through (80, 0) and (0, 60).

d Same intersection as in **b** $(100.8, 16.8)$ $H = 201.6$

Objective line passes through (120, 0) and (0, 20).

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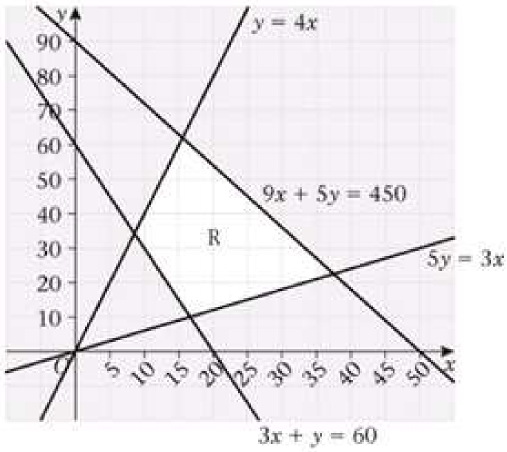
Exercise C, Question 3

Question:



The diagram shows a feasible region, R.
Find the optimal point and the optimal value, using

a the vertex testing method, with the objective 'minimise $J = x + 4y$',

b the vertex testing method, with the objective 'maximise $K = x + y$',

c the objective line method, with the objective 'minimise $L = 6x + y$',

d the objective line method, with the objective 'maximise $M = 2x + y$'.

Solution:

a Need intersection of $3x + y = 60$ and $5y = 3x$

$\left(16\frac{2}{3}, 10\right)$ $J = 56\frac{2}{3}$

b Need intersection of $y = 4x$ and $9x + 5y = 450$

$\left(15\frac{15}{29}, 62\frac{2}{29}\right)$ $K = 77\frac{17}{29}$

c Need intersection of $3x + y = 60$ and $y = 4x$

Objective line passes through (10, 0) and (0, 60).

$\left(8\frac{4}{7}, 34\frac{2}{7}\right)$ $L = 85\frac{5}{7}$

d Need intersection of $9x + 5y = 450$ and $5y = 3x$

Objective line passes through (40, 0) and (0, 80).

$(37.5, 22.5)$ $m = 97.5$

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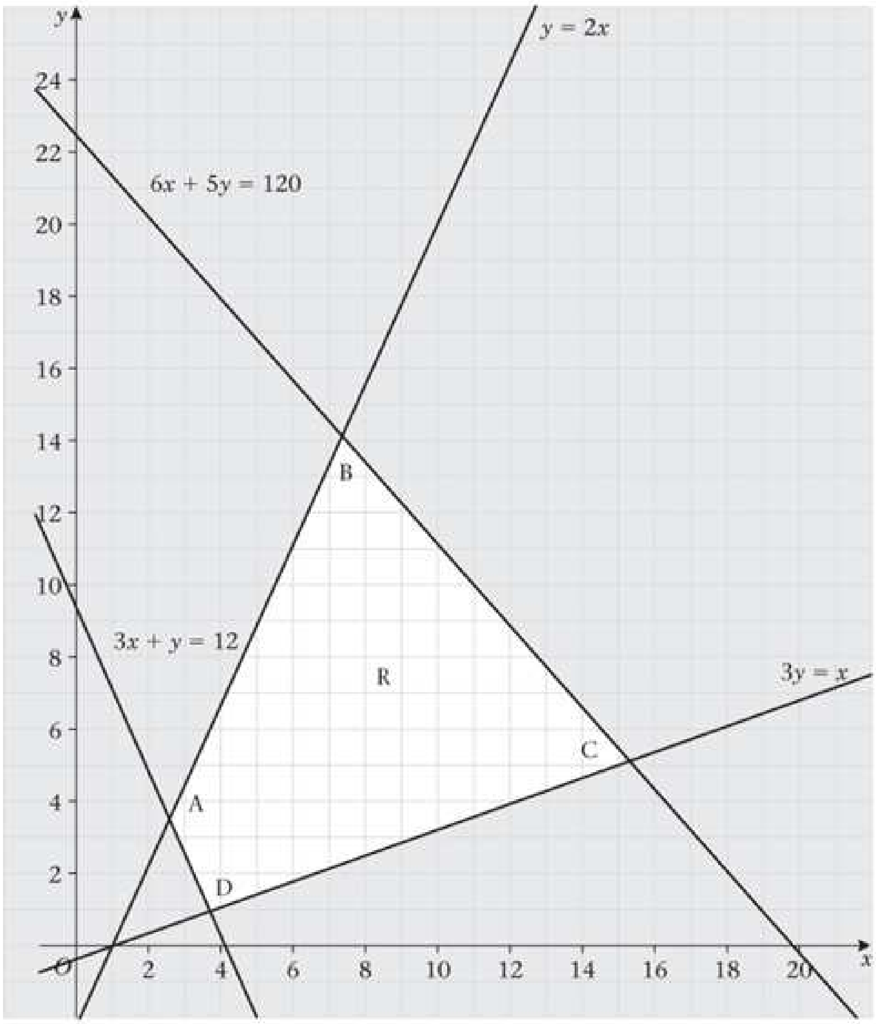
Exercise C, Question 4

Question:



The diagram shows a feasible region, R, which is defined by

$3x + y \geq 12$

$y \leq 2x$

$3y \geq x$

$6x + 5y \leq 120.$

Determine which vertex, A, B, C or D, is the optimal point for each of the following objectives.

a maximise x
b minimise x
c maximise y
d minimise y
e maximise $6x+y$
f minimise $6x+y$
g maximise $2x+5y$
h minimise $2x+5y$
i maximise $3x+2y$
j minimise $3x+2y$.

Solution:

a C

b A

c B

d D

e C

f A

g B

h D

i C

j D

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Exercise C, Question 5

Question:

A linear programming problem is given as
minimise $C = 3x + 2y$

subject to $2x + y \geq 160$

$x + y \geq 120$

$x + 3y \geq 180$

and $x \geq 0, y \geq 0$

a Draw a graph to illustrate the feasible, R.
(Take the values $0 \leq x \leq 180$ and $0 \leq y \leq 160$ for your axes.)

b Use the vertex testing method, on the four vertices, to identify the optimal point and optimal value.

Given that the objective changes to
minimise $C_1 = 2x + 3y$,

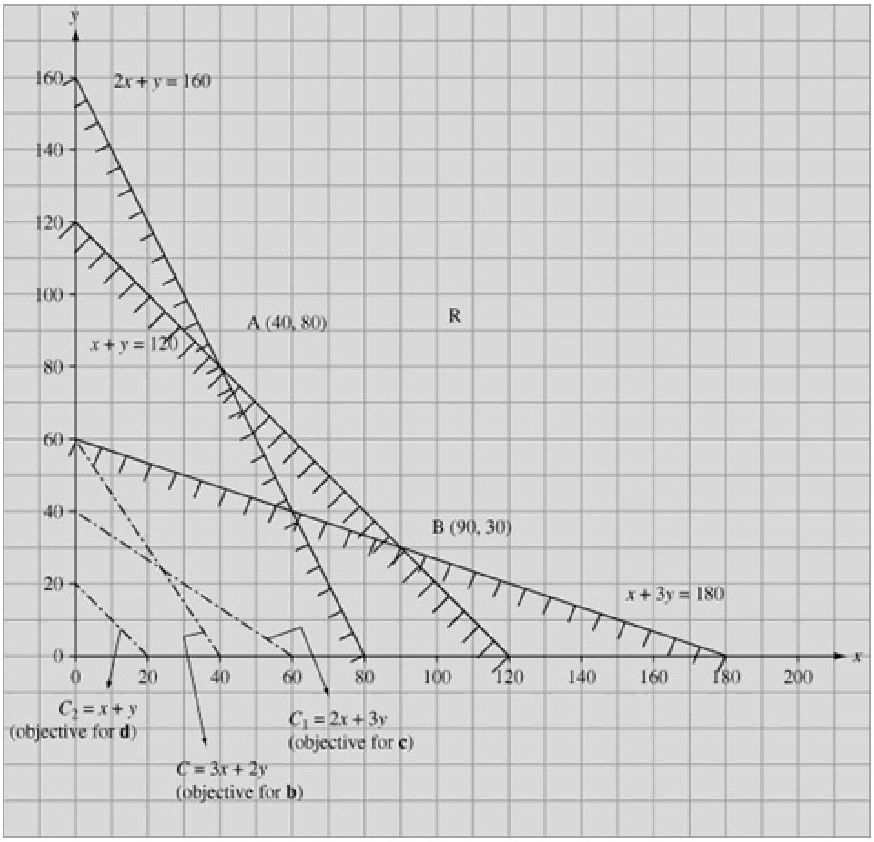
c draw a suitable objective line and use it to identify the optimal point and optimal value.

Given that the objective changes to
minimise $C_2 = x + y$,

d explain why there is more than one solution to the problem.

Solution:

a



b

Vertices	$C = 3x + 2y$
(0, 160)	320
(40, 80)	280
(90, 30)	330
(180, 0)	540

so minimum is (40, 80) value of $C = 280$

c (90, 30) $C_1 = 270$

d C_2 is parallel to $x + y = 120$ so all points from A to B are optimal points.

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Exercise C, Question 6

Question:

A feasible region, R, is defined by

$y \leq 5x$

$14x + 9y \leq 630$

$2y \geq x$

$4x + y \geq 60$

a Draw a graph to illustrate the feasible region, R.
(Take the values $0 \leq x \leq 45$ and $0 \leq y \leq 70$ for your axes.)

b Use the objective line method to determine the optimal point and the optimal value given the objective

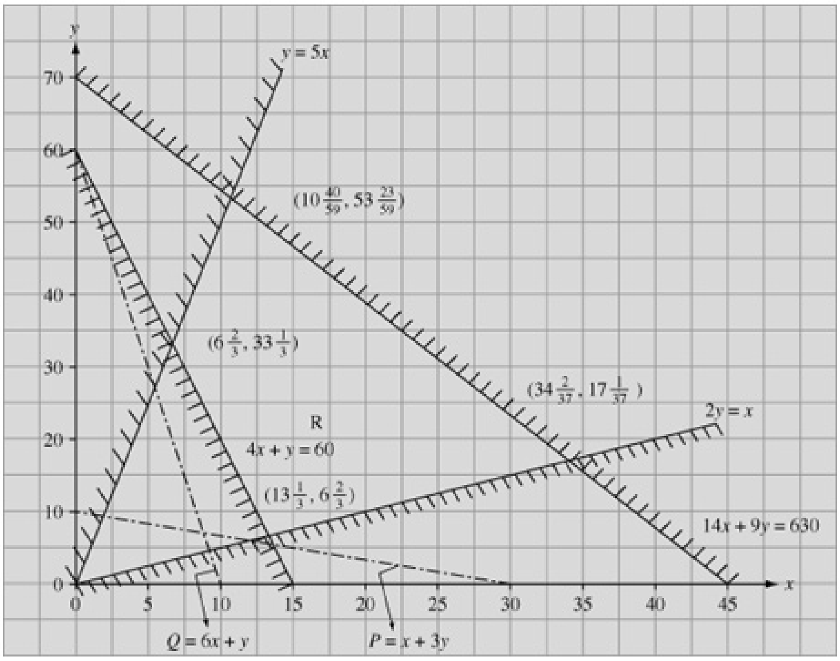
i minimise $P = x + 3y$,

ii maximise $Q = 6x + y$.

In each case you must draw, and label, an objective line, and find *exact* values.

Solution:

a



b **i** $\left(13\frac{1}{3}, 6\frac{2}{3}\right)$ $P = 33\frac{1}{3}$

ii $\left(34\frac{2}{37}, 17\frac{1}{37}\right)$ $Q = 221\frac{13}{37}$

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Exercise C, Question 7

Question:

A feasible region, R, is defined by

$y \geq 10x$

$x \leq 120$

$2y - x \geq 100$

$2x + y \leq 400$

a Draw a graph to illustrate R.

Given that the objective function is $z = 5x + y$,

b determine the exact value of z, if z is to be

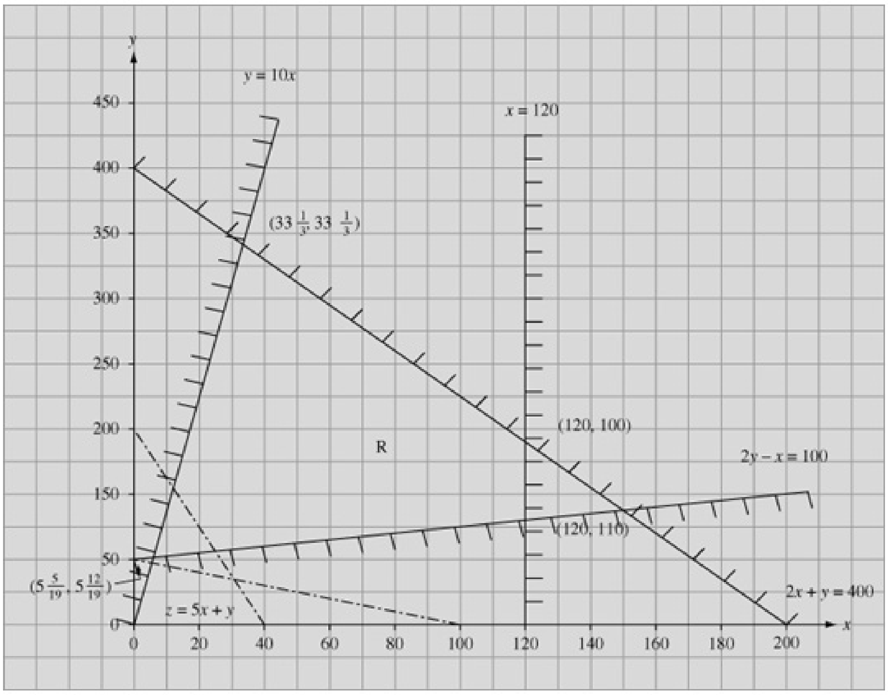
i maximised,

ii minimised.

c Determine the optimal value of $x + 2y$. Give your answer as an exact value.

Solution:

a



b **i** $(120, 160)$ $z = 760$

ii $\left(5\frac{5}{19}, 52\frac{12}{19}\right)$ $z = 78\frac{18}{19}$

c Optimal point $\left(33\frac{1}{3}, 333\frac{1}{3}\right)$ optimal value of $x + 2y = 700$

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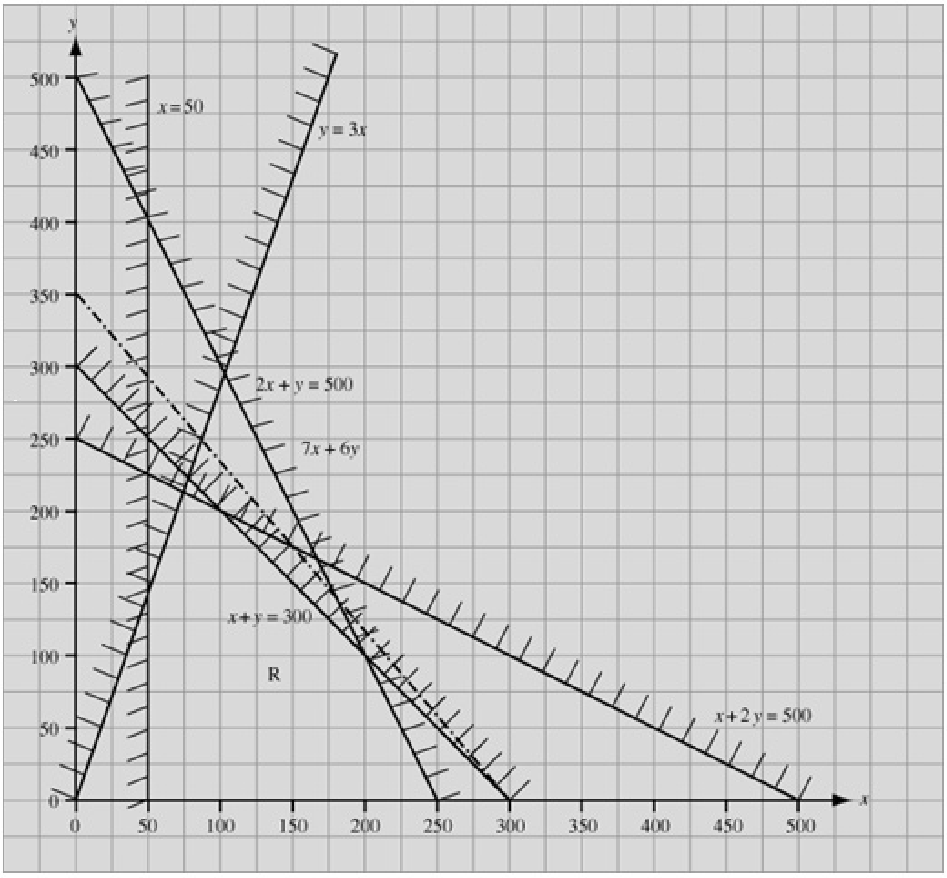
Exercise C, Question 8

Question:

Solve the linear programming problem posed in Exercise 6A, Question 5 (page 120).

Solution:



Objective line passes through (0, 350) and (300, 0). Maximum point is (200, 100).
P = 2000.

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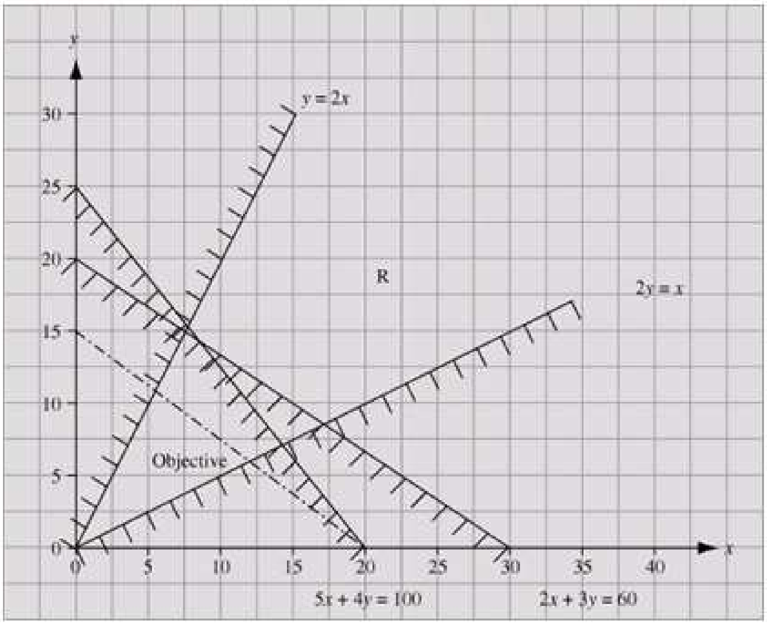
Exercise C, Question 9

Question:

Solve the linear programming problem posed in Exercise 6A, Question 7.

Solution:



Objective line passes through (0, 15) and (20, 0).

Intersection of $2x + 3y = 60$

$5x + 4y = 100$

$\left(8\frac{4}{7}, 14\frac{2}{7}\right)$

$\text{value} = 8285\frac{5}{7}$

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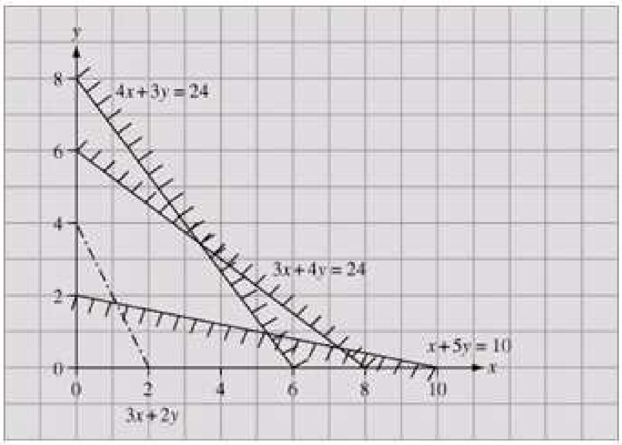
Exercise D, Question 1

Question:

Solve the following linear programming problems, given that integer values are required for the decision variables.

$$\begin{aligned} \text{Maximise } & 3x + 2y \\ \text{subject to } & x+5y \geq 10 \\ & 3x+4y \leq 24 \\ & 4x+3y \leq 24 \\ & x,y \geq 0 \end{aligned}$$

Solution:



Maximum integer value (5, 1)
$3x+2y=17$

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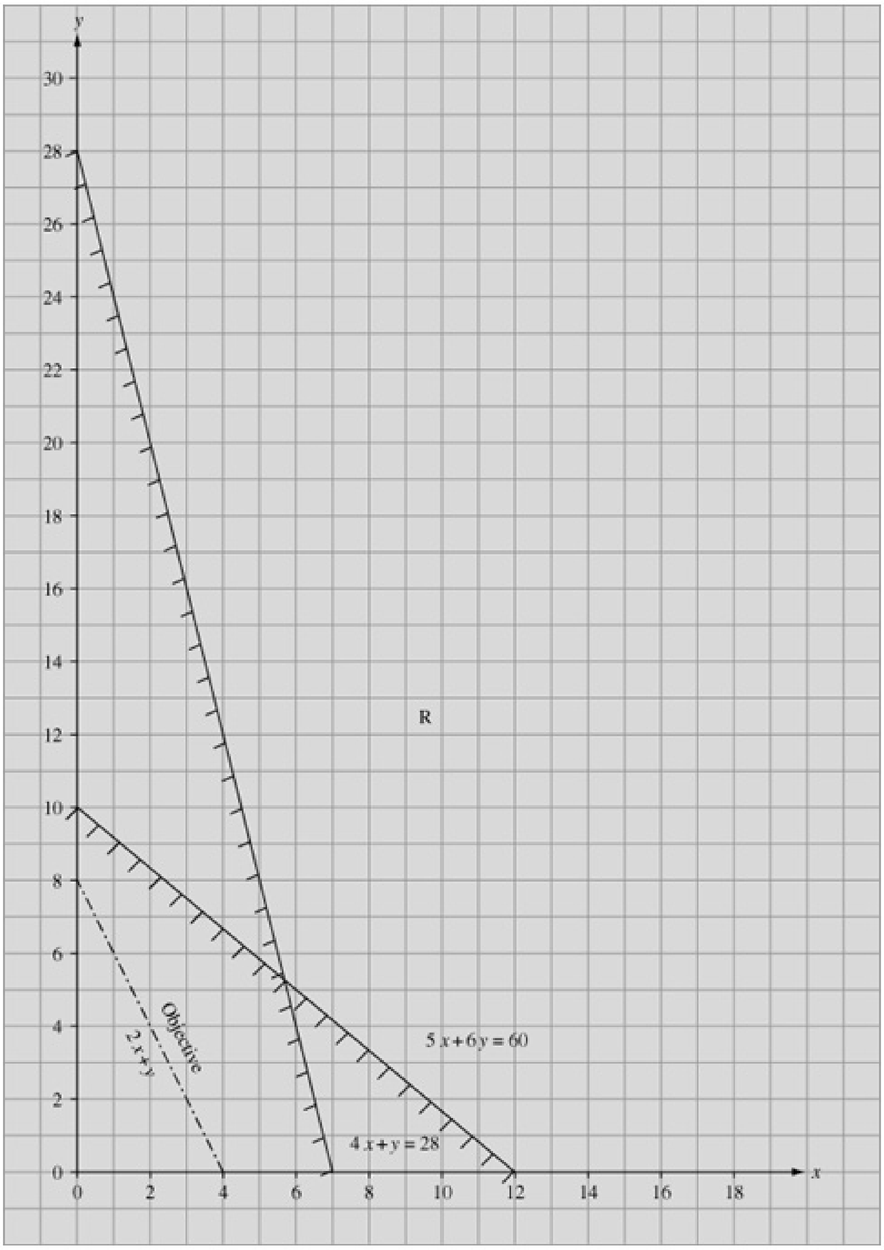
Exercise D, Question 2

Question:

Solve the following linear programming problems, given that integer values are required for the decision variables.

$$\begin{aligned} \text{Minimise } & 2x + y \\ \text{subject to } & 5x + 6y \geq 60 \\ & 4x + y \geq 28 \\ & x, y \geq 0 \end{aligned}$$

Solution:



Minimum integer values (6, 6)

$2x + y = 18$

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Exercise D, Question 3

Question:

Solve the following linear programming problem, given that integer values are required for the decision variables.

Maximise $5x + 2y$

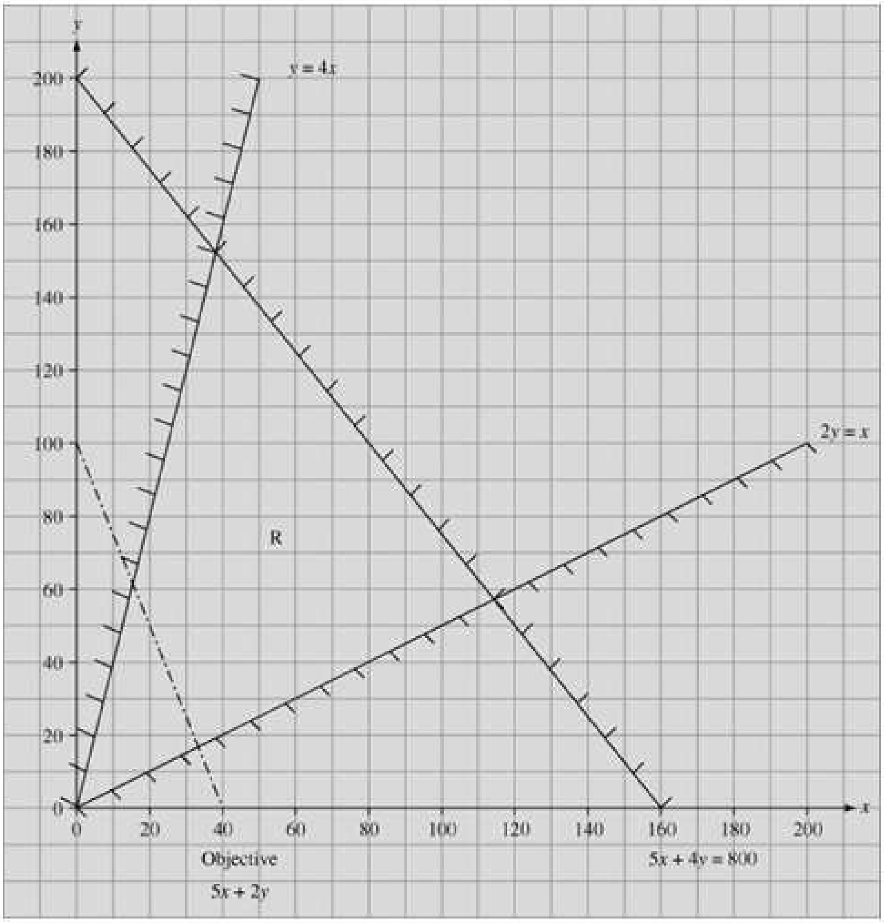
subject to $2y \geq x$

$5x + 4y \leq 800$

$y \leq 4x$

$x, y \geq 0$

Solution:



Solving $2y = x$ and $5x + 4y = 800$ simultaneously gives $\left(114\frac{2}{7}, 57\frac{1}{7}\right)$

Test integer values nearby.

Point	$2y \geq x$	$5x+4y \leq 800$	$5x+2y$
(114, 57)	✓	✓	684
(114, 58)	✓	X	–
(115, 57)	X	X	–
(115, 58)	✓	X	–

so optimal point is (114, 57) value 684

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Exercise D, Question 4

Question:

Maximise $2x + y$

subject to $3x + 5y \le 1500$

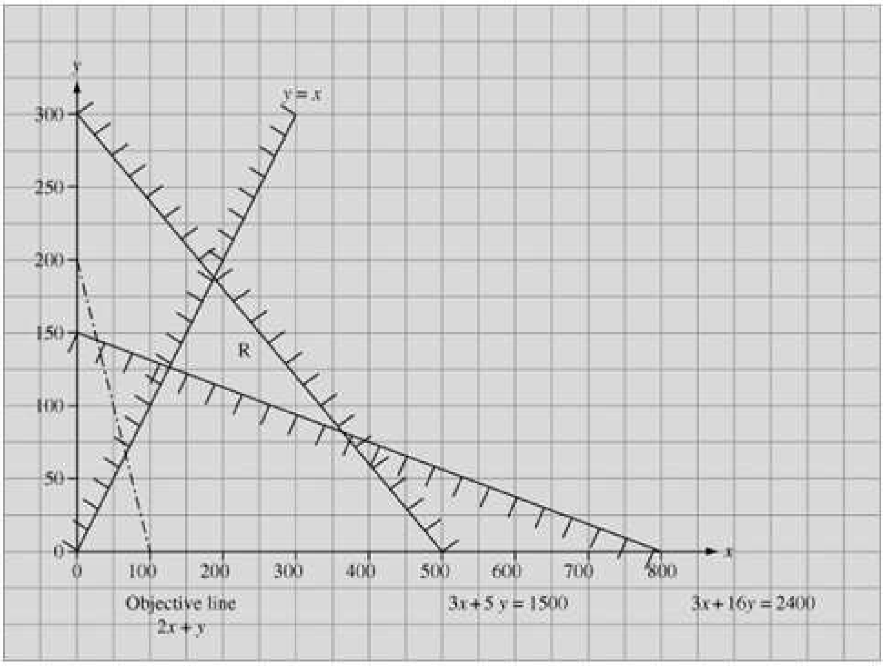
$3x + 16y \ge 2400$

$y \le x$

$x, y \ge 0$

Solve this problem.

Solution:



Solving $3x+16y=2400$
$3x+5y=1500$
simultaneously gives
$\left(363\frac{7}{11}, 81\frac{9}{11}\right)$

Taking integer point

Point	$3x+16y \geq 2400$	$3x+5y \leq 1500$	$2x+y$
(363, 81)	✓	X	–
(363, 82)	✓	✓	808
(364, 81)	✓	X	–
(364, 82)	X	✓	–

So optimal integer point is (363, 82)

value 808

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Exercise D, Question 5

Question:

Solve this problem.

A chocolate manufacturer is producing two hand-made assortments, gold and silver, to commemorate 50 years in business.

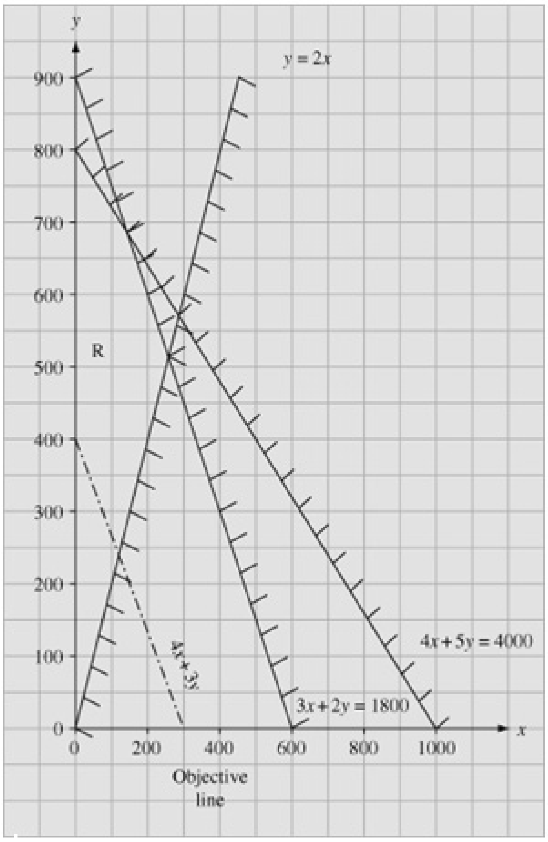
It will take 30 minutes to make all the chocolates for one box of gold assortment and 20 minutes to make the chocolates for one box of silver assortment.

It will take 12 minutes to wrap and pack the chocolates in one box of gold assortment and 15 minutes for one box of silver assortment.

The manufacturer needs to make at least twice as many silver as gold assortments. The gold assortment will be sold at a profit of 80p, and the silver at a profit of 60p.

There are 300 hours available to make the chocolates and 200 hours to wrap them. Maximise the profit, P.

Solution:



Intersection of $4x+5y=4000$ and $3x+2y=1800$

giving $\left(142\frac{6}{7}, 685\frac{5}{7}\right)$

Testing nearby integer points

Point	$4x+5y \le 4000$	$3x+2y \le 1800$	$80x+60y$
(142, 685)	√	√	52460
(142, 686)	√	√	52520
(143, 685)	√	√	52540
(143, 686)	X	X	

so maximum integer solution is 52540 pennies at (143, 685).

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Exercise D, Question 6

Question:

Solve this problem.

A floral display is required for the opening of a new building. The display must be at least 30 m long and is to be made up of two types of planted displays, type A and type B.

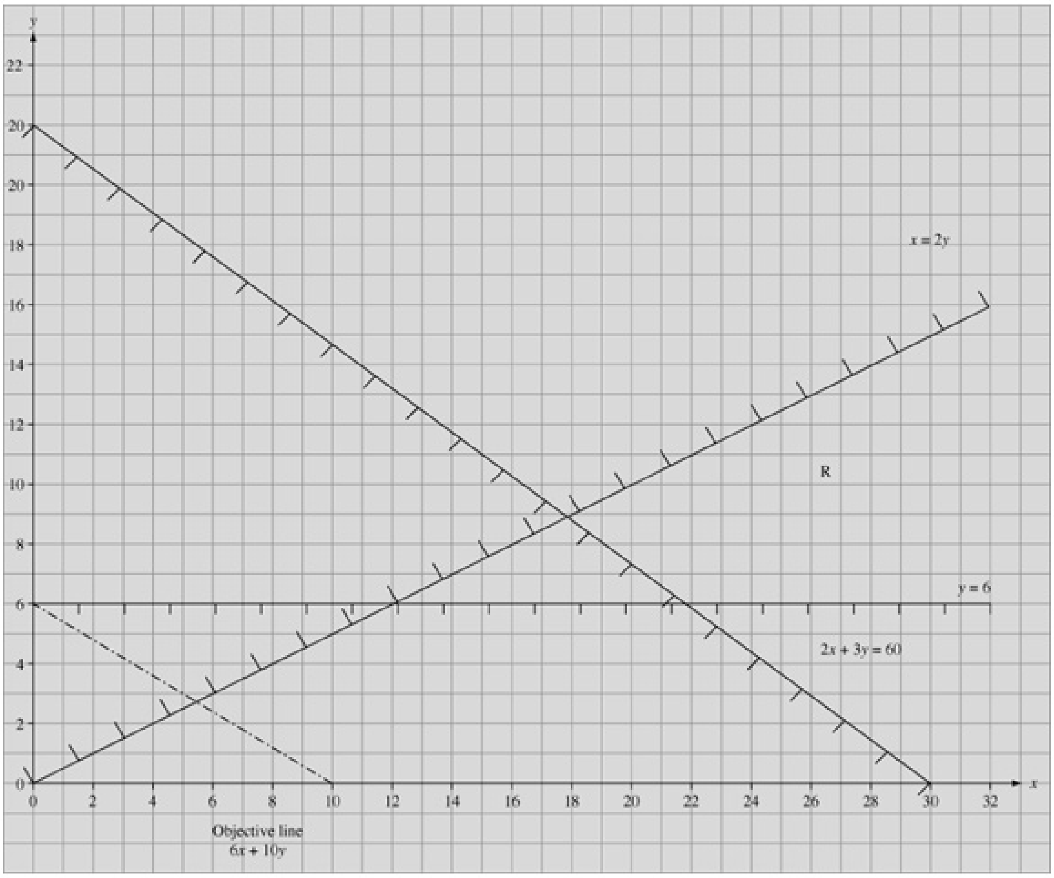
Type A is 1 m in length and costs £6

Type B is 1.5 m in length and costs £10

The client wants at least twice as many type A as type B, and at least 6 of type B.

The cost is to be minimised.

Solution:



Intersection of $2x+3y=60$ and $y=6$

(21, 6)

$\text{cost} = 6x+10y$ so minimum cost $= 6\times 21+60=$ £186

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Exercise D, Question 7

Question:

Solve this problem.

A toy company makes two types of board game, Cludopoly and Trivscrab. As well as the board each game requires playing pieces and cards.

The company uses two machines, one to produce the pieces and one to produce the cards.

Both machines can only be operated for up to ten hours per day.
The first machine takes 5 minutes to produce a set of pieces for Cludopoly and 8 minutes to produce a set of pieces for Trivscrab.

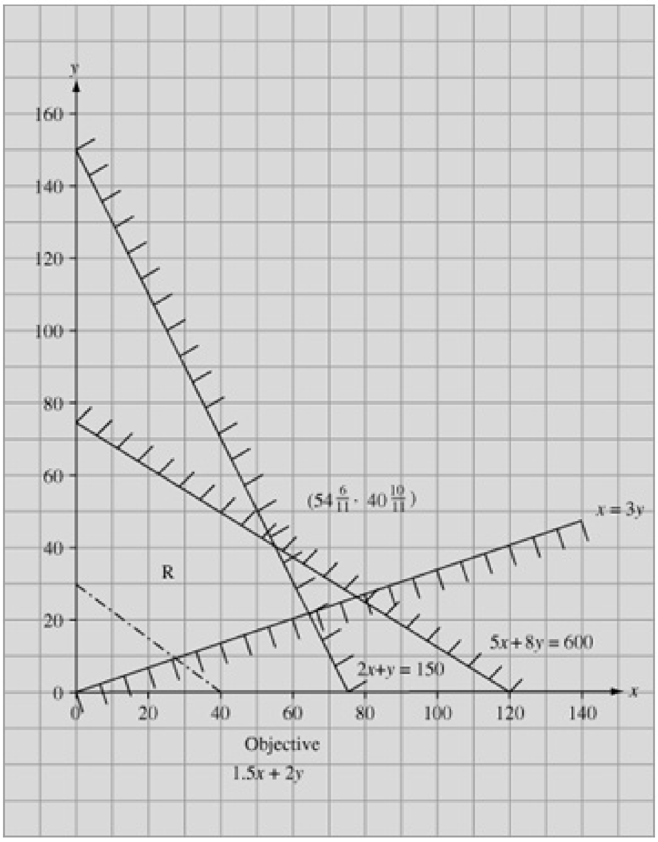
The second machine takes 8 minutes to produce a set of cards for Cludopoly and 4 minutes to produce a set of cards for Trivscrab.

The company knows it will sell at most three times as many games of Cludopoly as Trivscrab.

The profit made on each game of Cludopoly is £1.50 and £2.50 on each game of Trivscrab.

The company wishes to maximise its daily profit.

Solution:



Intersection of $5x+8y=600$

$2x+y=150$ giving $\left(54\frac{6}{11}, 40\frac{10}{11}\right)$

Points	$5x+8y \leq 600$	$2x+y \leq 150$	$1.5x+2y$
(54, 40)	√	√	161
(54, 41)	√	√	163
(55, 40)	√	√	162.5
(55, 41)	X	X	

so maximum value is 163 at (54, 41).

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Exercise D, Question 8

Question:

Solve this problem.

A librarian needs to purchase bookcases for a new library. She has a budget of £3000 and 240 m^2 of available floor space. There are two types of bookcase, type 1 and type 2, that she is permitted to buy.

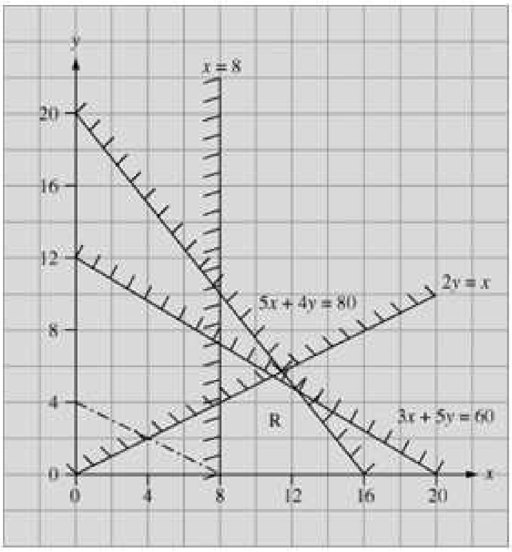
Type 1 costs £150, needs 15 m^2 of floor space and has 40 m of shelving.

Type 2 costs £250, needs 12 m^2 of floor space and has 50 m of shelving.

She must buy at least 8 type 1 bookcases and wants at most $\frac{1}{3}$ of all the bookcases to be type 2.

She wishes to maximise the total amount of shelving.

Solution:



Intersection of $3x+5y=60$

$5x+4y=80$ giving $\left(12\frac{4}{13}, 4\frac{8}{13}\right)$

Points	$3x+5y\leq 60$	$5x+4y\leq 80$	$40x+60y$
(12, 4)	✓	✓	720
(12, 5)	X	✓	–
(13, 4)	✓	X	–
(13, 5)	X	X	–

Maximum value is 720 at (12, 4).

Solutionbank D1
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Exercise E, Question 1

Question:

Mr Baker is making cakes and fruit loaves for sale at a charity cake stall. Each cake requires 200 g of flour and 125 g of fruit. Each fruit loaf requires 200 g of flour and 50 g of fruit. He has 2800 g of flour and 1000 g of fruit available.

Let the number of cakes that he makes be x and the number of fruit loaves he makes be y.

a Show that these constraints can be modelled by the inequalities $x+y \leq 14$ and $5x+2y \leq 40$.

Each cake takes 50 minutes to cook and each fruit loaf takes 30 minutes to cook. There are 8 hours of cooking time available.

b Obtain a further inequality, other than $x \geq 0$ and $y \geq 0$, which models this time constraint.

c On graph paper illustrate these three inequalities, indicating clearly the feasible region.

d It is decided to sell the cakes for £3.50 each and the fruit loaves for £1.50 each. Assuming that Mr Baker sells all that he makes, write down an expression for the amount of money P, in pounds, raised by the sale of Mr Baker's products.

e Explaining your method clearly, determine how many cakes and how many fruit loaves Mr Baker should make in order to maximise P. ***E***

Solution:

a *flour*: $200x + 200y \leq 2800$

so $x + y \leq 14$

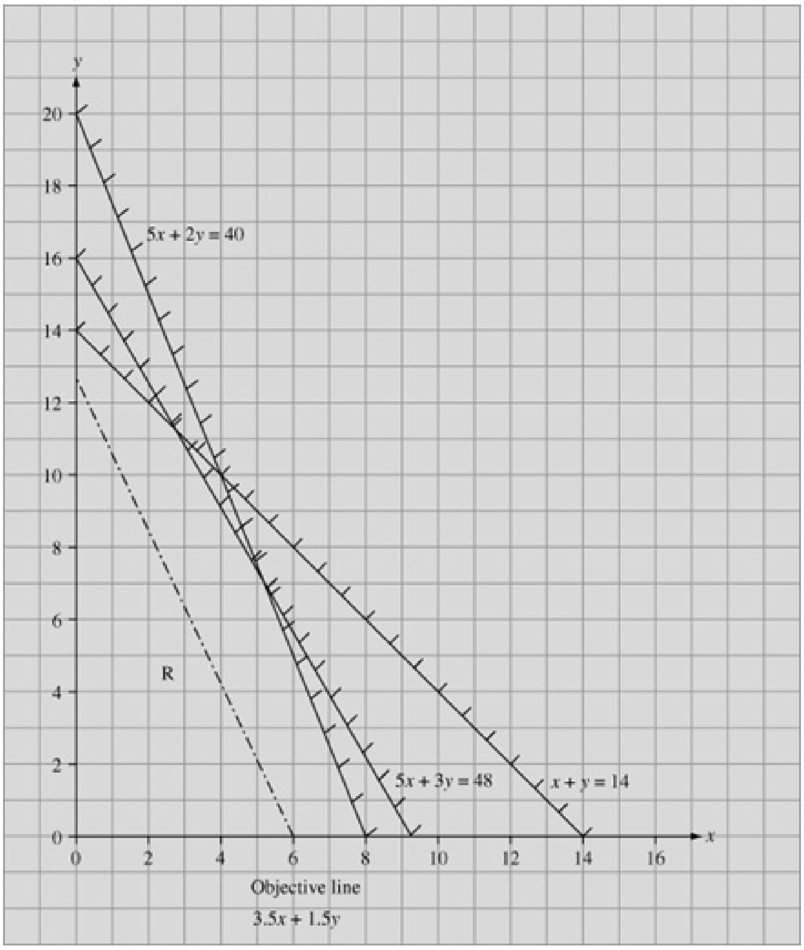
fruit: $125x + 50y \leq 1000$

so $5x + 2y \leq 40$

b Cooking time $50x + 30y \leq 480$

so $5x + 3y \leq 48$

c



d $P = 3.5x + 1.5y$

e Integer solution required (6, 5)

$P_{max} = £28.50$

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Exercise E, Question 2

Question:

A junior librarian is setting up a music recording lending section to loan CDs and cassette tapes. He has a budget of £420 to spend on storage units to display these items.
Let x be the number of CD storage units and y the number of cassette storage units he plans to buy.
Each type of storage unit occupies $0.08\,\text{m}^3$, and there is a total area of $6.4\,\text{m}^3$ available for the display.

a Show that this information can be modelled by the inequality
$x + y \leq 80$

The CD storage units cost £6 each and the cassette storage units cost £4.80 each.

b Write down a second inequality, other than $x \geq 0$ and $y \geq 0$, to model this constraint.

The CD storage unit displays 30 CDs and the cassette storage unit displays 20 cassettes.

The chief librarian advises the junior librarian that he should plan to display at least half as many cassettes as CDs.

c Show that this implies that $3x \leq 4y$.

d On graph paper, display your three inequalities, indicating clearly the feasible region.

The librarian wishes to maximise the total number of items, T, on display. Given that
$T = 30x + 20y$

e determine how many CD storage units and how many cassette storage units he should buy, briefly explaining your method. ***E***

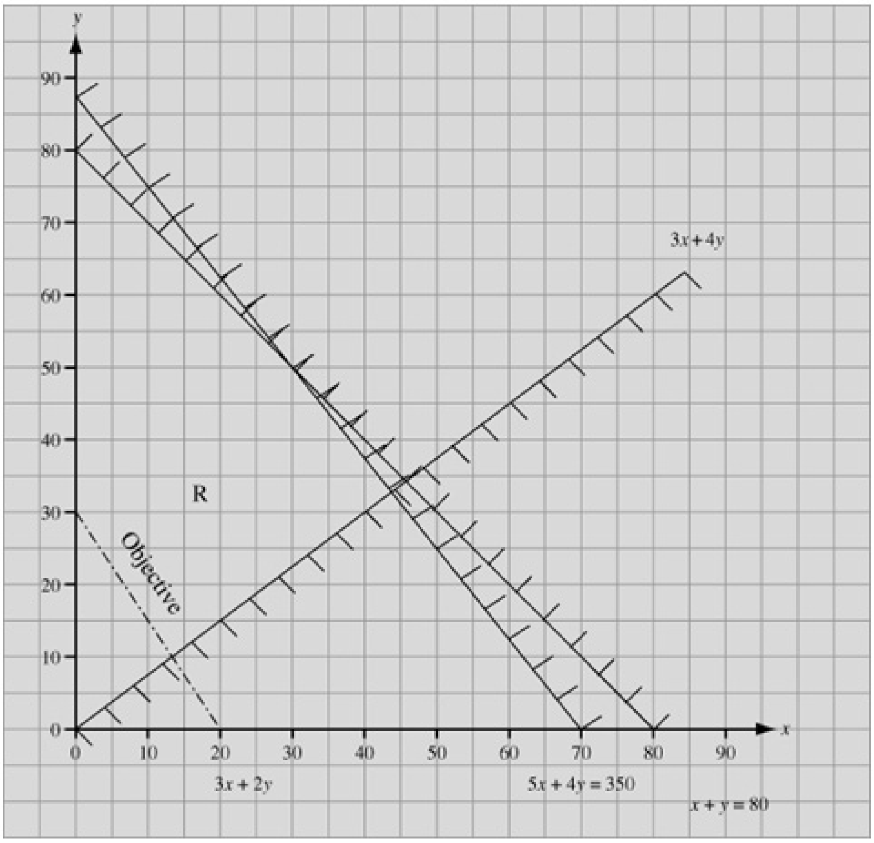
Solution:

a storage: $0.08x + 0.08y \le 6.4$
so $x + y \le 80$

b cost: $6x + 4.8y \le 420$ so $5x + 4y \le 350$

c Display $30x \le 2 \times 20y$
$3x \le 4y$

d



e Integer solution required (43, 33).
He should buy 43 CD storage units and 33 cassette storage units.

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Exercise E, Question 3

Question:

The headteacher of a school needs to hire coaches to transport all the year 7, 8 and 9 pupils to take part in the recording of a children's television programme. There are 408 pupils to be taken and 24 adults will accompany them on the coaches. The headteacher can hire either 54 seater (large) or 24 seater (small) coaches. She needs at least two adults per coach. The bus company has only seven large coaches but an ample supply of small coaches.

Let x and y be the number of large and small coaches hired respectively.

a Show that the situation can be modelled by the three inequalities:
   i $9x + 4y \geq 72$
   ii $x + y \leq 12$
   iii $x \leq 7$.

b On graph paper display the three inequalities, indicating clearly the feasible region.

A large coach costs £336 and a small coach costs £252 to hire.

c Write down an expression, in terms of x and y, for the total cost of hiring the coaches.

d Explain how you would locate the best option for the headteacher, given that she wishes to minimise the total cost.

e Find the number of large and small coaches that the headteacher should hire in order to minimise the total cost and calculate this minimum total cost.

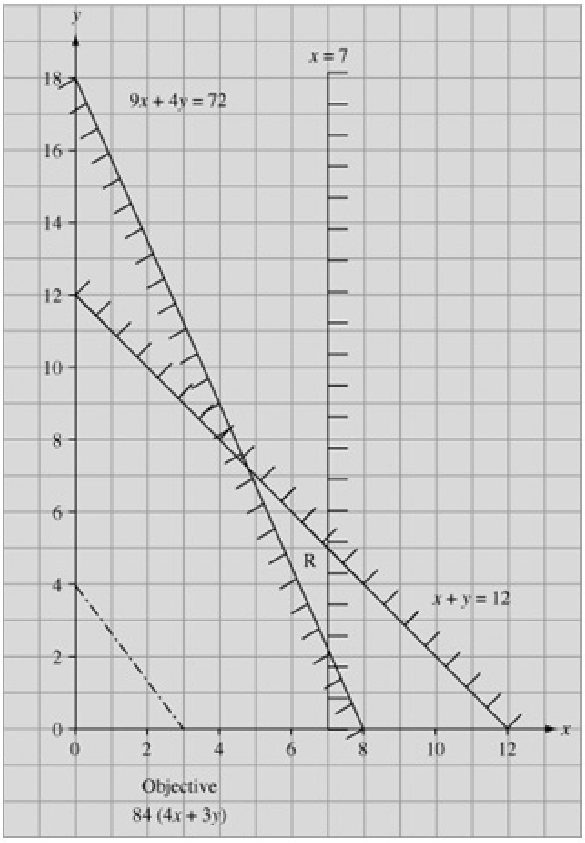
Solution:

a **i** Total number of people $54x+24y \geq 432$ so $9x+4y \geq 72$

ii number of adults $x+y \leq 12$

iii number of large coaches $x \leq 7$

b



c minimise $C = 336x+252y$

$= 84(4x+3y)$

d Objective line passes through (0, 4) (3, 0)

e Integer coordinates needed (7, 3) so hire 7 large coaches and 3 small coaches
cost = £ 3108

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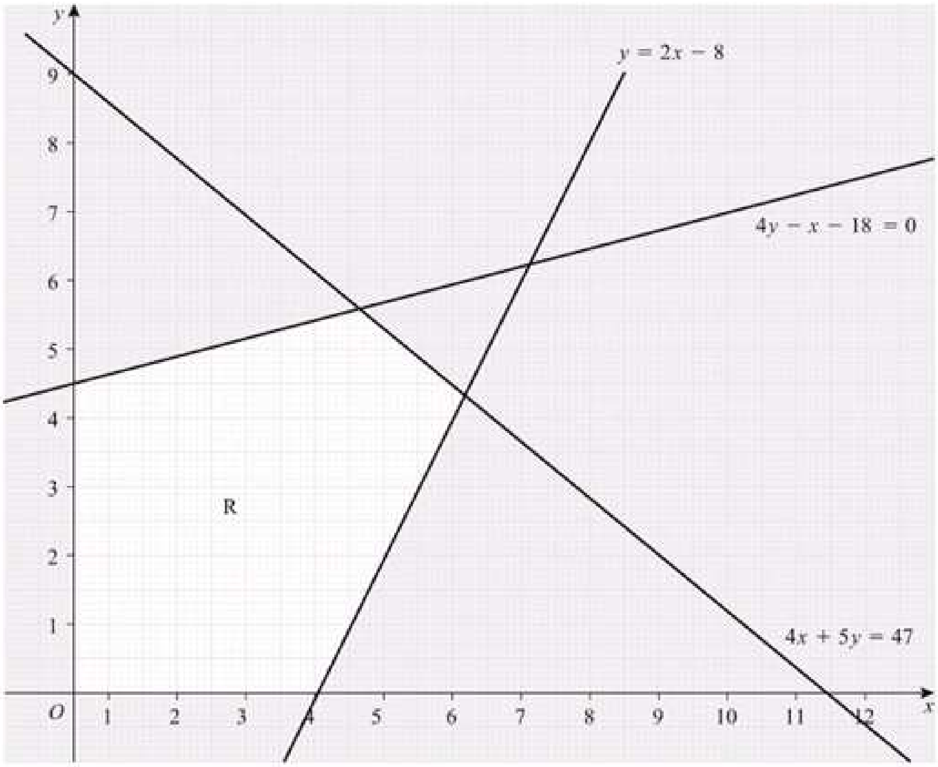
Exercise E, Question 4

Question:

The graph below was drawn to solve a linear programming problem. The feasible region, R, includes the points on its boundary.



a Write down the inequalities that define the region R.
The objective function, P, is given by $P = 3x + 2y$.

b Find the value of x and the value of y that lead to the maximum value of P. Make your method clear.

c **i** Give an example of a practical linear programming problem in which it would be necessary for the variables to have integer values.

ii Given that the solution must have integer values of x and y, find the values that lead to the maximum value of P.

Solution:

a $4x + 5y \leq 47$

$y \geq 2x - 8$

$4y - x - 8 \leq 0$

$x, y \geq 0$

b Solving simultaneous equations $y = 2x - 8$

$4x + 5y = 47$

$\left(6\frac{3}{14}, 4\frac{3}{7}\right)$

c **i** For example where x and y

- types of car to be hired
- number of people, etc.

ii $(6, 4)$

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Exercise E, Question 5

Question:

A company produces plates and mugs for local souvenir shops. The plates and mugs are manufactured in a two-stage process. Each day there are 300 minutes available for the completion of the first stage and 400 minutes available for the completion of the second stage. In addition the mugs require some hand painting. There are 150 minutes available each day for hand painting.

Product	Stage 1	Stage 2	Hand painting
Plate	$2\frac{1}{2}$	5	—
Mug	3	2	2

The above table shows the production time, in minutes, required for the plates and the mugs.

All plates and mugs made are sold. The profit on each plate sold is £2 and the profit on each mug sold is £4. The company wishes to determine how many plates and mugs to make so as to maximise its profits each day.

Let x be the number of plates made and y the number of mugs made each day.

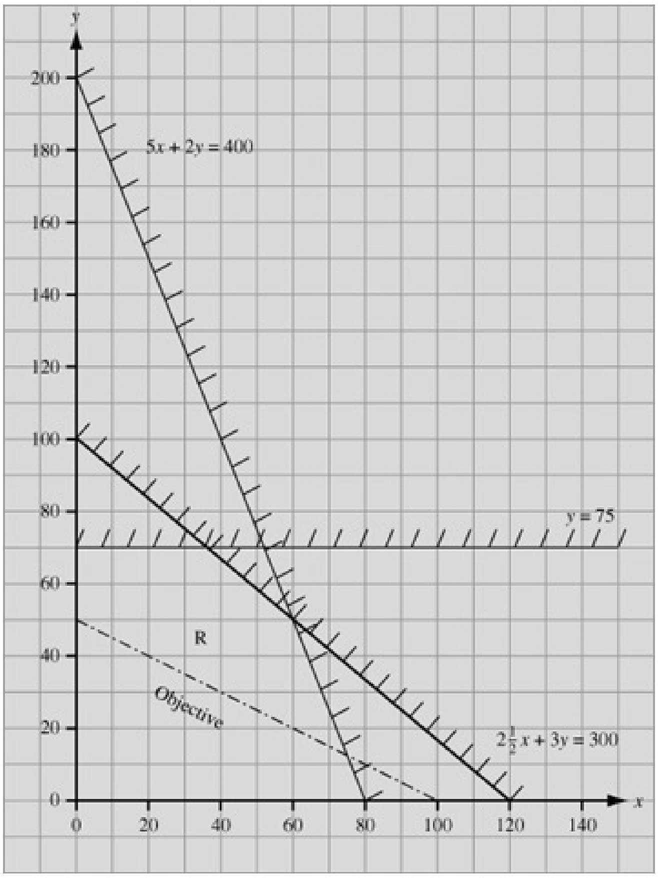
a Write down the three constraints, other than $x \geq 0, y \geq 0$, satisfied by x and y.

b Write down the objective function to be maximised.

c Using the graphical method, solve the resulting linear programming problem. Determine the optimal number of plates and mugs to be made each day and the resulting profit.

d When the optimal solution is adopted determine which, if any, of the stages has available time which is unused. State the amount of unused time. ***E***

Solution:



a $2\frac{1}{2}x+3y \le 300 \quad [5x+6y \le 600]$

$5x+2y \le 400$

$2y \le 150 \quad [y \le 75]$

b Maximise $P=2x+4y$

c $(30, 75)$ $P=360$

d The optimal point is at the intersection of $y=75$ and $2\frac{1}{2}x+3y=300$.

So the constraint $5x+2y \le 400$ is not at its limit.

At $(30, 75)$ $5x+2y=300$ so 100 minutes are unused.

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Exercise E, Question 6

Question:

Two fertilizers are available, a liquid X and a powder Y. A bottle of X contains 5 units of chemical A, 2 units of chemical B and $\frac{1}{2}$ unit of chemical C. A packet of Y contains 1 unit of A, 2 units of B and 2 units of C. A professional gardener makes her own fertilizer. She requires at least 10 units of A, at least 12 units of B and at least 6 units of C. She buys x bottles of X and y packets of Y.

a Write down the inequalities which model this situation.

b Construct and label the feasible region.

A bottle of X costs £2 and a packet of Y costs £3.

c Write down an expression, in terms of x and y, for the total cost £T.

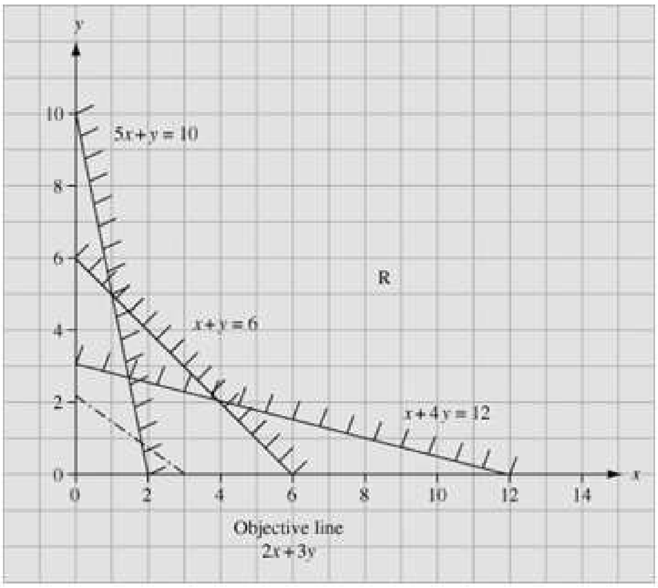
d Using your graph, obtain the values of x and y that give the minimum value of T. Make your method clear and calculate the minimum value of T.

e Suggest how the situation might be changed so that it could no longer be represented graphically.

Solution:

a Chemical A $5x + y \geq 10$
Chemical B $2x + 2y \geq 12$ $[x + y \geq 6]$
Chemical C $\frac{1}{2}x + 2y \geq 6$ $[x + 4y \geq 12]$
$x, y \geq 6$

b



c $T = 2x + 3y$

d (4, 2) $T = 14$

e If there were 3 or more variables the problem could not be solved graphically.
So adding a third fertiliser z, would mean a graphical method could not be used.

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Exercise A, Question 1

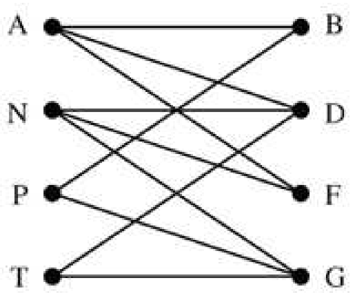
Question:

Four inspectors, Alan, Nicola, Philip and Trudy are to inspect four supermarket departments: bakery, delicatessen, fish and grocery.

Alan is qualified to inspect bakery, delicatessen and fish.
Nicola is qualified to inspect delicatessen, fish and grocery.
Philip is qualified to inspect bakery and grocery.
Trudy is qualified to inspect delicatessen and grocery.

Draw a bipartite graph to show this information.

Solution:



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Exercise A, Question 2

Question:

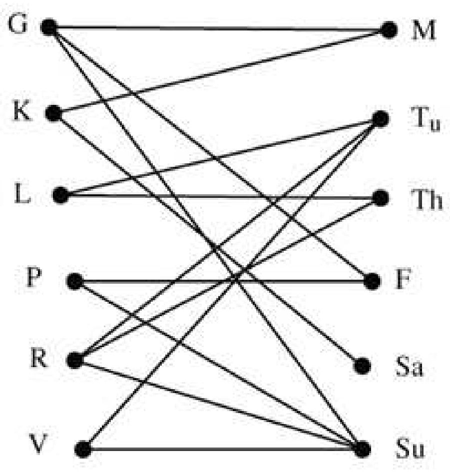
Tours of the local caves are given daily by six guides: Graham, Keith, Lethna, Preety, Rob, and Vicky. The caves are closed on Wednesdays but guides are needed for Monday, Tuesday, Thursday, Friday, Saturday and Sunday.

Next week the guides are available on the days shown in the following table.

Guide	Days available
Graham	Monday, Friday, Sunday
Keith	Monday, Saturday
Lethna	Tuesday, Thursday
Preety	Sunday, Friday
Rob	Tuesday, Sunday, Thursday
Vicky	Tuesday, Sunday

Represent this information on a bipartite graph.

Solution:



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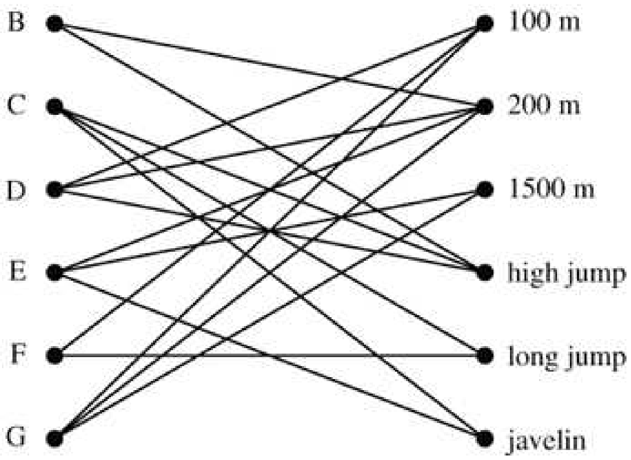
Exercise A, Question 3

Question:

An athletics team, Bill, Charley, Dara, Eun Jung, Fred and Gopan, will compete in six events: 100 m, 200 m, 1500 m, high jump, long jump and javelin. Each person may only compete in one event.

Bill prefers the high jump or 200 m.
Charley does not like the running events, but is happy to compete in either of the jumping events or the javelin.
Dara will do the 100 m, 200 m or the high jump.
Eun Jung likes competing in 200 m, 1500 m or javelin.
Fred prefers long jump or 100 m.
Gopan will do any of the running events 100 m, 200 m or 1500 m.
Draw a bipartite graph to represent this information.

Solution:



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Exercise B, Question 1

Question:

The tour director of a museum needs to allocate five of his guides to parties of tourists from France, Germany, Italy, Japan and Spain. The table shows the languages spoken by the five guides.

Ruth	French	Spanish	
Steve	German	Japanese	
Tony	French	German	
Ursula	Spanish	Italian	
Victoria	Italian	Spanish	Japanese

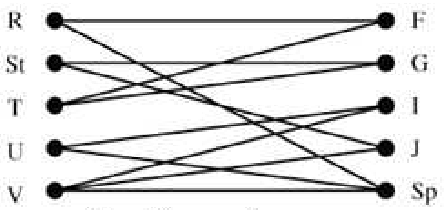
a Draw a bipartite graph to model this situation.

The director allocates Ruth, Steve, Ursula and Victoria to the parties who speak the first language in their individual lists.

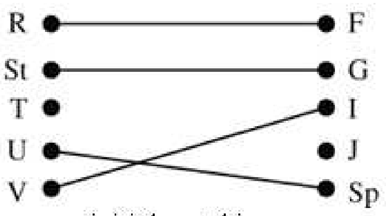
b Starting from this matching use the maximum matching algorithm to find a complete matching. Indicate clearly how the algorithm has been applied in this case. ***E***

Solution:

a



bipartite graph



initial matching

either
Alternating path: T – G=St – J
Change states T=G – St=J
Complete matching

R = F St = J T = G U = SP V = I

or
Alternating path: T – F = R – Sp = U – I = V – J
Change status T = F – R = Sp – U = I – V = J
complete matching

R = Sp St = G T = F U = I V = J

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Exercise B, Question 2

Question:

In order to help new A-level students to select their courses a college organises an open evening. Some students already studying A-level courses have agreed to talk about one of their A-level courses. Six of these students, Ann, Barry, David, Gemma, Jasmine and Nickos, are between them following six A-level courses in Chemistry, English, French, History, Mathematics and Physics.

The table below shows the courses being followed by each student:

Ann	English	French	History
Barry	History	English	
David	French	Chemistry	Mathematics
Gemma	Physics	Mathematics	History
Jasmine	Mathematics	Physics	Chemistry
Nickos	English	Mathematics	French

a Draw a bipartite graph to model this situation.

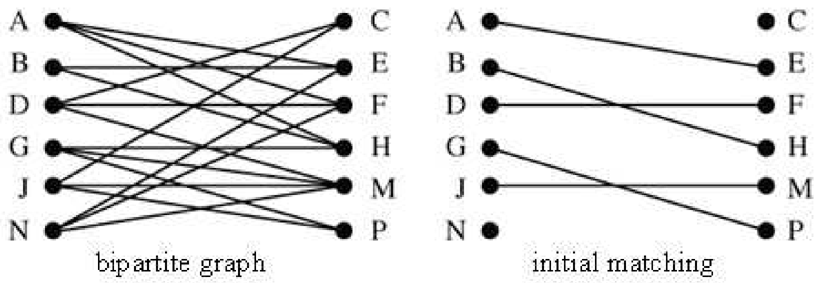
Initially Ann, Barry, David, Gemma and Jasmine are allocated to the first subject in their lists.

b Starting from this matching use the maximum matching algorithm to find a complete matching. Indicate clearly how the algorithm has been applied in this case.

c Explaining your reasoning carefully, determine whether or not your answer to **b** is unique.

Solution:

a



b Six possible alternating paths could be used, (one only is needed!)

i N – F = D – C
ii N – M = J – C
iii N – E = A – F = D – C
iv N – F = D – M = J – C
v N – E = A – F = D – M = J – C
vi N – M = J – P = G – H = B – E = A – F = D – C

Change status to give the complete matching

Person	Path (i)	Path (ii)	Path (iii)	Path (iv)	Path (v)	Path (vi)
A	E	E	F	E	F	F
B	H	H	H	H	H	E
D	C	F	C	M	M	C
G	P	P	P	P	P	H
J	M	C	M	C	C	P
N	F	M	E	F	E	M

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Exercise B, Question 3

Question:

Five coach drivers, Mihi, Pat, Robert, Sarah and Tony, have to be assigned to drive five coaches for the following school trips:

Adupgud Senior School is going to the Lake District
Brayknee Junior School is going to the seaside
Korry Stur Junior School is going to a concert
Learnalott Senior School is going to the museum (two coaches needed)

Mihi and Sarah would like to drive senior school children. Robert and Pat would like to go on the seaside trip. Pat and Tony would like to attend the concert. Robert and Pat would like to visit the museum. Pat and Tony would like to visit the Lake District.

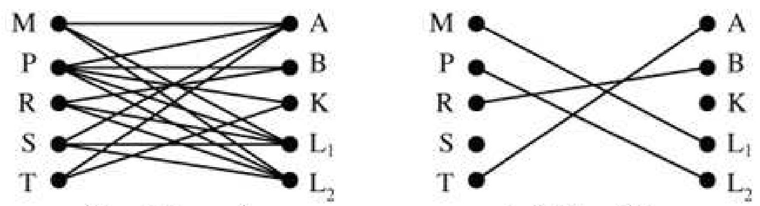
The driver manager wishes to assign each driver to a trip they would like to do.

a Draw a bipartite graph to show the trips that the drivers would like to make.
Initially Mihi and Pat are assigned to Learnalott Senior School, Robert is assigned to Brayknee Junior School and Tony is assigned to Adupgad Senior School.

b Starting from this matching use the maximum matching algorithm to find a complete matching. You must indicate clearly how the algorithm has been applied in this case. State your alternating path and the final matching. ***E***

Solution:

a



bipartite graph initial matching

b Four possible alternating paths – one only is needed

i S – L = P – K

ii S – L = M – A = T – K

iii S – A = T – K

iv S – L = P – A = T – K

change status to give a complete matching

Person	Path (i)	Path (ii)	Path (iii)	Path (iv)
M	L	A	L	L
P	K	L	L	A
R	B	B	B	B
S	L	L	A	L
T	A	K	K	K

Solutionbank D1
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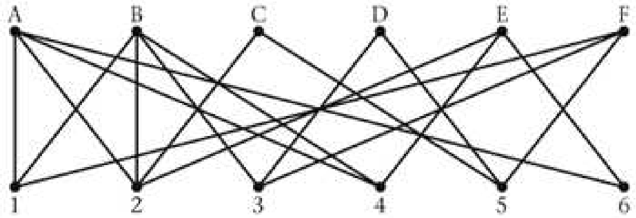
Exercise B, Question 4

Question:



The bipartite graph above shows a mapping between six volunteers, A, B, C, D, E and F, and six tasks, 1, 2, 3, 4, 5 and 6. The lines indicate which tasks each volunteer is qualified to do.

The initial matching is $A-2, B-1, C-5, D-3$ and $E-4$.

a Starting from this matching, use the maximum matching algorithm to find a complete matching. You must indicate clearly how the algorithm has been applied in this case. State your alternating path and your final matching.

Volunteer E now insists on doing task 2.

b State the changes that need to be made to the initial model to accommodate this. ***E***

Solution:

a 12 possible alternating paths – one only is needed

i F – 1 = B – 2 = A – 6
ii F – 1 = B – 2 = A – 4 = E – 6
iii F – 1 = B – 3 = D – 5 = C – 2 = A – 6
iv F – 1 = B – 3 = D – 5 = C – 2 = A – 4 = E – 6
v F – 1 = B – 4 = E – 6
vi F – 1 = B – 4 = E – 2 = A – 6
vii F – 5 = C – 2 = A – 1 = B – 4 = E – 6
viii F – 5 = C – 2 = A – 4 = E – 6
ix F – 5 = C – 2 = A – 6
x F – 3 = D – 5 = C – 2 = A – 6
xi F – 3 = D – 5 = C – 2 = A – 4 = E – 6

xii F – 3 = D – 5 = C – 2 = A – 1 = B = 4 – E = 6

Change status to give complete matching

Volunteer	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)
A	6	4	6	4	2	6	1	4	6	6	4	1
B	2	2	3	3	4	4	4	1	1	1	1	4
C	5	5	2	2	5	5	2	2	2	2	2	2
D	3	3	5	5	3	3	3	3	3	5	5	5
E	4	6	4	6	6	2	6	6	4	4	6	6
F	1	1	1	1	1	1	5	5	5	3	3	3

b Remove E and 2 and all arcs attached to each of them. Then run the algorithm as usual on the reduced problem.

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Exercise B, Question 5

Question:

At a school fete six teachers, A, B, C, D, E and F, are to run six stalls, R, S, T, U, V and W.

A would prefer to run T but is willing to run R
B would prefer to run S but is willing to run R or W
C would prefer to run U but is willing to run S
D would prefer to run V but is willing to run R
E is willing to run T or V
F is willing to run V

a Draw a bipartite graph to model this situation.

Initially, A, B, C and D are matched to their preferred choices.

b Indicate this initial matching in a distinctive way on the bipartite graph drawn in **a**.

c Use the maximum matching algorithm to find a maximum matching, listing clearly your alternating path.

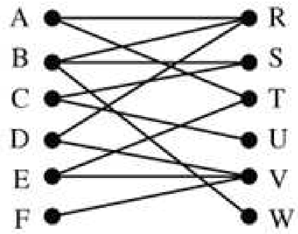
d Explain why it is not possible to find a complete matching. You should make specific reference to individual stalls and teachers.

Teacher A now offers to run stall S.

e Draw a new bipartite graph. Hence, using the previous initial matching and the maximum matching algorithm, determine if it is now possible to obtain a complete matching. If it is possible, give the matching, stating clearly your alternating path; if it is still not possible explain why.

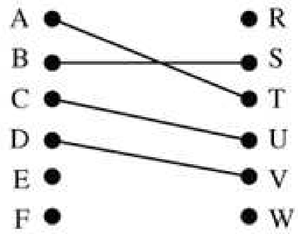
Solution:

a



bipartite graph

b



initial matching

c There are 3 alternating paths, that could be used

i E – V = D – R or
ii E – T = A – R or
iii F – V = D – R

Change status to give

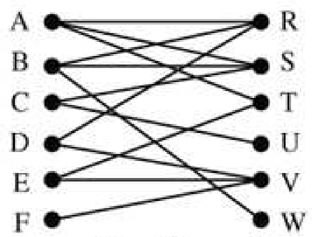
i E = V – D = R or
ii E = T – A = R or
iii F = V – D = R

Maximum matching

Person	Path (i)	Path (ii)	Path (iii)
A	T	R	T
B	S	S	S
C	U	U	U
D	R	V	R
E	V	T	?
F	?	?	V

d For example C must do U and B must do W, but B and C are the only people who can do S. So a complete matching is not possible.

e



new bipartite graph

Depending on the path chosen in **c**

either E – V = D – R and F – V = E – T = A – S = B – W

or E – T = A – S = B – W and F – V = D – R

or E – T = A – S = B – R and F – V = D – R = B – W

Final matching
A = S B = W C = U E = T D = R F = V

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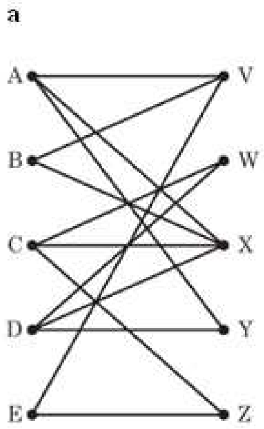
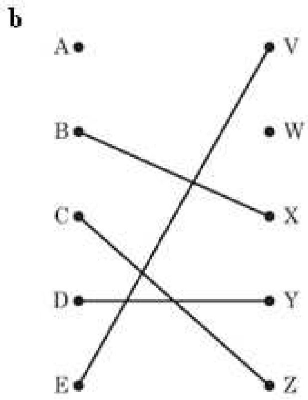
Exercise B, Question 6

Question:





Five people A, B, C, D and E are to be matched to five tasks V, W, X, Y and Z. A bipartite graph showing the possible matching is shown in **a**, and an initial matching is shown in **b**.

There are a number of distinct alternating paths that can be generated from this initial matching. Two such paths are

A – Y – D – W and A – X – B – V – E – Z – C – W.

a Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

b Using the maximum matching algorithm and the given initial matching,

i find two further distinct alternating paths, making your reasoning clear,

ii write down the complete matchings they generate. *E*

Solution:

a A = Y B = X C = Z D = W E = V
A = X B = V C = W D = Y E = Z

b

V = E– Z = C – W
A
X = B – V = E – Z = C – W (given)
W (given)
Y = D
X = B – V = E – Z = C – W

i Changing status in the two new paths
A = V – E = Z – C = W and
A = Y – D = X – B = V – E = Z – C = W

ii Giving matchings
A = V B = X C = W D = Y E = Z and
A = Y B = V C = W D = X E = Z

Solutionbank D1

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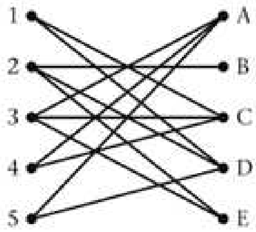
Exercise B, Question 7

Question:

A college has five vacant jobs, A, B, C, D and E. There are five applicants who are labelled 1, 2, 3, 4 and 5. The applicants are only qualified for certain jobs and the following table summarises this information. The diagram shows a bipartite graph modelling this information.

Applicant	Jobs qualified for
1	C, D
2	B, D, E
3	A, C, E
4	A, C
5	A, D



Initially Applicant 2 is allocated to job B, 3 is attached to C and 5 is allocated to A.

a **i** Show this matching clearly on a diagram.

ii Starting from this matching, use the maximum matching algorithm to obtain an improved matching. State clearly your alternating path and show this improved matching on Diagram 2.

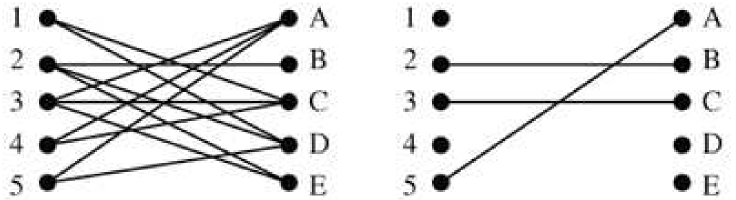
iii Hence obtain a complete matching. State clearly your alternating path and this complete matching.

The interviewing committee decides that applicant 3 is to be appointed to job C.

b If this appointment is made, explain why it is not possible to fill all the other jobs with the remaining applicants. ***E*** (adapted)

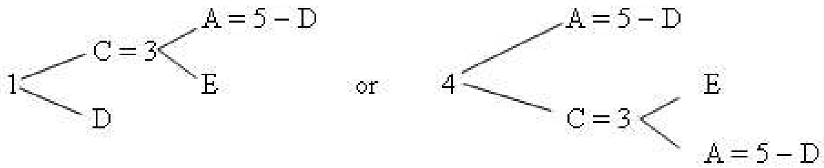
Solution:

a **i**



bipartite graph initial matching

ii



So six possible alternating paths – one only needs to be used changing status

i 1 = D
ii 1 = C – 3 = E
iii 1 = C – 3 = A – 5 = D
iv 4 = A – 5 = D
v 4 = C – 3 = E
vi 4 = C – 3 = A – 5 = D

Giving improved matchings

Applicant	(i)	(ii)	(iii)	(iv)	(v)	(vi)
1	D	C	C	?	?	?
2	B	B	B	B	B	B
3	C	E	A	C	E	A
4	?	?	?	A	C	C
5	A	A	D	D	A	D

iii The next path depends on the previous path choice.

Previously chosen path	Possible next alternating path
i	4 – C = 3 – E
ii	4 – C = 1 – D
iii	4 – C = 1 – D = 5 – A = 3 – E
iv	1 – C = 3 – E
v	1 – D
vi	1 – D = 5 – A = 3 – E

Changing status leads to the following complete matching

Applicant	(i)	(ii)	(iii)	(iv)	(v)	(vi)
1	D	D	D	C	D	D
2	B	B	B	B	B	B
3	E	E	E	E	E	E
4	C	C	C	A	C	C
5	A	A	A	D	A	A

b If 3 matches with C
2 must now do E since no one else can but 2 is the only person who can do B too so a complete matching is not possible.

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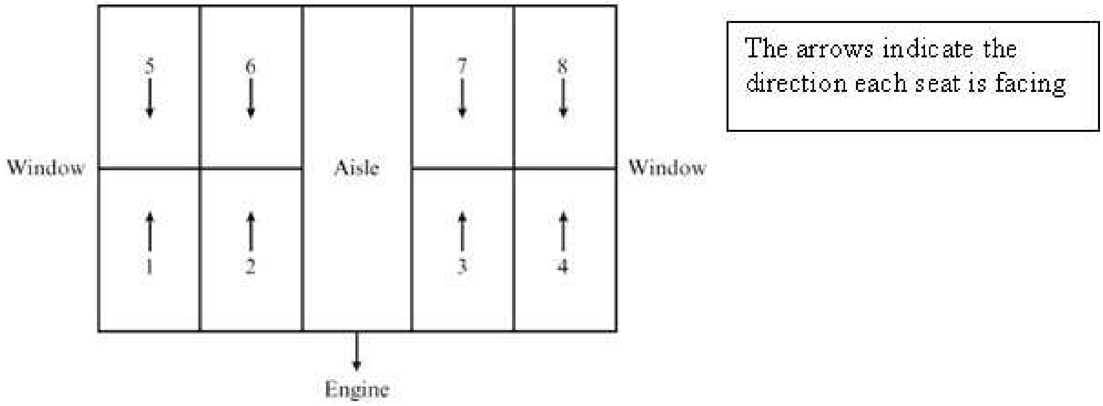
Exercise B, Question 8

Question:



The diagram above represents eight seats in a railway carriage, numbered 1, 2, 3, 4, 5, 6, 7 and 8. These are the last eight seats available on a special sightseeing trip. The booking clerk has to arrange the seating for the final customers. Six customers make the following requests:

Ms A wants an aisle seat facing the engine (6 or 7)
Mr B wants a window seat (1, 4, 5 or 8)
Rev C wants a seat with his back to the engine (1, 2, 3 or 4)
Mrs D wants an aisle seat (2, 3, 6 or 7)
Miss E wants a seat facing the engine (5, 6, 7 or 8)
Dr F wants a window seat with her back to the engine (1 or 4)

Initially the clerk assigns the seats as follows:

A to 6, B to 5, C to 4, D to 2, E to 7 and F to 1
The day before departure Mr and Mrs G join the trip. They ask to sit next to each other (1 and 2, 3 and 4, 5 and 6 or 7 and 8). The clerk reassigns the seats, using as far as possible the original seat assignments as the initial matching.

a Choose two seats for Mr and Mrs G and, using a bipartite graph, model the possible seat allocations of the other customers.

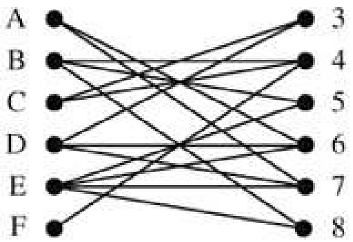
b Indicate, in a distinctive way, those elements of the clerk's original matching that are still possible.

c Using your answer to part **b** as the initial matching, apply the maximum matching algorithm. You must state your alternating path and your final maximum matching. ***E***

Solution:

Mr + Mrs G must be assigned to two seats next to each other.
1+2 or 3+4 or 5+6 or 7+8
Depending on what you change we get

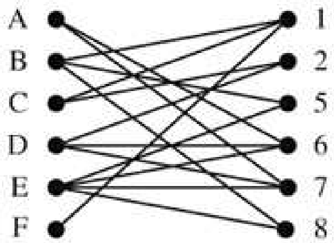
Alternative 1 (1 and 2 assigned to Mr + Mrs G)



Initially
A = 6
B = 5
C = 4
E = 7
For example, alternating paths

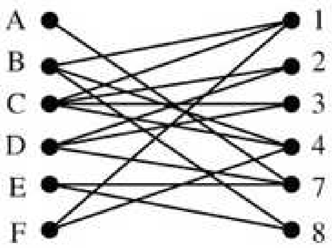
D − 7 = E − 8 and F − 4 = C − 3
Change status D = 7 − E = 8 and F = 4 − C = 3
A = 6 C = 3 E = 8
B = 5 D = 7 F = 4
Mr + Mrs G in 1 and 2

Alternative 2 (3+4 assigned to Mr + Mrs G)



Initially
A = 6
B = 5
D = 2
E = 7
For example, alternating path
C − 2 = D − 7 = E − 8
Change status C = 2 − D = 7 − E = 8
A = 6 C = 2 E = 8
B = 5 D = 7 F = 1
Mr + Mrs G in 3 and 4

Alternative 3 (5 and 6 assigned to Mr + Mrs G)



Initially

$C = 4$

$D = 2$

$E = 7$

$F = 1$

For example, alternating path

$A - 7 = E - 8$ and $B - 4 = C - 3$

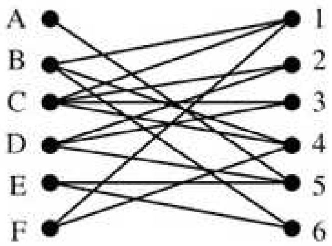
Change status $A = 7 - E = 8$ and $B = 4 - C = 3$

$A = 7 \quad C = 3 \quad E = 8$

$B = 4 \quad D = 2 \quad F = 1$

Mr + Mrs G in 5 and 6

Alternative 4 (7 and 8 assigned to Mr + Mrs G)



Initially

$A = 6$

$B = 5$

$C = 4$

$D = 2$

$F = 1$

For example, alternating path

$E - 5 = B - 4 = C - 3$

Change status $E = 5 - B = 4 - C = 3$

$A = 6 \quad C = 3 \quad E = 5$

$B = 4 \quad D = 2 \quad F = 1$

Mr + Mrs G in 7 and 8

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1 Review Exercise
Exercise A, Question 1

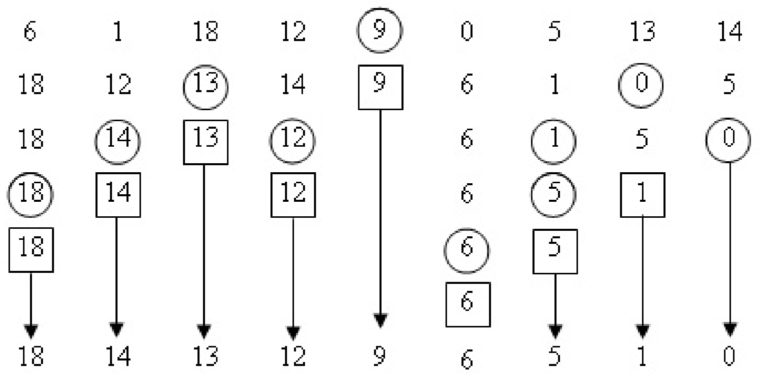
Question:

The table opposite shows the points obtained by each of the teams in a football league after they had each played 6 games. The teams are listed in alphabetical order. Carry out a quick sort to produce a list of teams in descending order of points obtained. ***E***

Ashford	6
Colnbrook	1
Datchet	18
Feltham	12
Halliford	9
Laleham	0
Poyle	5
Staines	13
Wraysbury	14

Solution:

For example,



Datchet (18), Wraysbury (14), Staines (13) Feltham (12), Halliford (9) Ashford (6), Poyle (5), Colnbrooke (1), Laleham (0)

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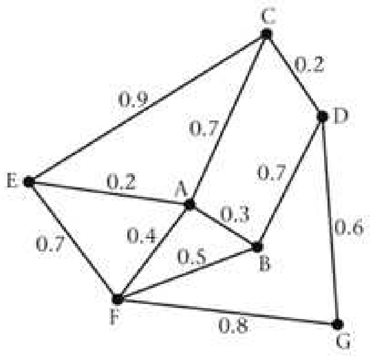
1 Review Exercise
Exercise A, Question 2

Question:



A local council is responsible for maintaining pavements in a district. The roads for which it is responsible are represented by arcs in the diagram. The road junctions are labelled A, B, C, ..., G. The number on each arc represents the length of that road in km.
The council has received a number of complaints about the condition of the pavements. In order to inspect the pavements, a council employee needs to walk along each road twice (once on each side of the road) starting and ending at the council offices at C. The length of the route is to be minimal. Ignore the widths of the roads.

a Explain how this situation differs from the standard route inspection problem.

b Find a route of minimum length and state its length. ***E***

Solution:

a All arcs are to be traversed twice, this is, in effect, repeating each arc. So all valencies are even.

b E.g. A–B–D–G–F–G–D–C–E–A–E–C–A–F–E–F–B–F–A–B–D–C–A
(all correct routes will have 23 letters in their name)
length $= 2 \times 6 = 12$ km

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1 Review Exercise
Exercise A, Question 3

Question:

a Use the binary search algorithm to try to locate the name *SABINE* in the following alphabetical list. Explain each step of the algorithm.

1	*ABLE*
2	*BROWN*
3	*COOKE*
4	*DANIEL*
5	*DOUBLE*
6	*FEW*
7	*OSBORNE*
8	*PAUL*
9	*SWIFT*
10	*TURNER*

b Find the maximum number of iterations of the binary search algorithm needed to locate a name in a list of 1000 names. ***E***

Solution:

a 1st pivot is $\left[\frac{1+10}{2}\right] \rightarrow 6$ FEW, SABINE after FEW

Rejecting: 1–6 list is now

7 OSBORNE
8 PAUL
9 SWIFT
10 TURNER

2nd pivot is $\left[\frac{7+10}{2}\right] \rightarrow 9$ SWIFT, SABINE before SWIFT.

Rejecting 9 and 10 list is now

7 OSBORNE
8 PAUL

3rd Pivot is $\left[\frac{7+8}{2}\right] \rightarrow 8$ PAUL, SABINE is after PAUL. Rejecting 7 and 8 list is now empty

So SABINE is not in the list

b Either:

maximum length of list remaining is

1000, 500, 250, 125, 63, 31, 15, 7, 3, 1

so 10 iterations.

or

we seek the smallest integer value of n such that

$$2^n > 1000$$

$$\log 2^n > \log 1000$$

$$n \log 2 > \log 1000$$

$$n > \frac{\log 1000}{\log 2}$$

$$n > 9.966$$

$\therefore n$ is 10

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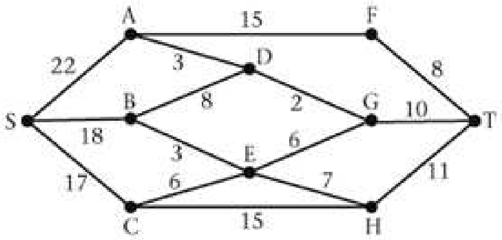
1 Review Exercise
Exercise A, Question 4

Question:



The diagram shows a network of roads.
The number on each edge gives the time, in minutes, to travel along that road.
Avinash wishes to travel from S to T as quickly as possible.

a Use Dijkstra's algorithm to find the shortest time to travel from S to T.

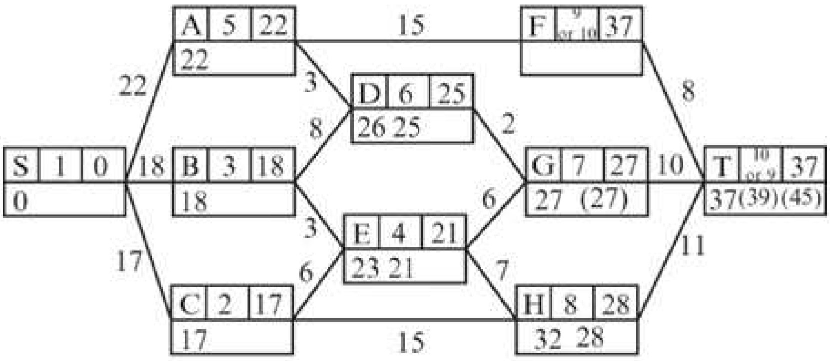
b Find a route for Avinash to travel from S to T in the shortest time. State, with a reason, whether this route is a unique solution.

On a particular day Avinash must include C in his route.

c Find a route of minimal time from S to T that includes C, and state its time. ***E***

Solution:

a



Shortest time: 37 minutes

b either S−A−D−G−T or S−B−E−G−T
The route is not unique; there are two of them (S−A−D−G−T and S−B−E−G−T)

c S−C−E−G−T length 39 minutes.

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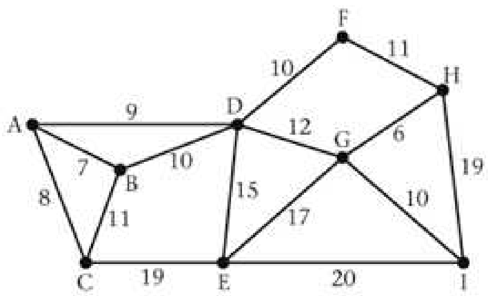
1 Review Exercise
Exercise A, Question 5

Question:

a State briefly
  i Prim's algorithm,
  ii Kruskal's algorithm.

b Find a minimum spanning tree for the network below using
  i Prim's algorithm, starting with vertex G,
  ii Kruskal's algorithm.

In each case write down the order in which you made your selection of arcs.



c State the weight of a minimum spanning tree.

d State, giving a reason for your answer, which algorithm is preferable for a large network. ***E***

Solution:

a **i** Prim

- Select any vertex to start the tree.
- Select the shortest arc that joins a vertex already in the tree to a vertex not yet in the tree. Add it to the tree.
- Continue selecting shortest arcs until all vertices are in the tree.

ii Kruskal

- Sort all arcs into ascending order of weight.
- Select the arc at least weight to start the tree.
- Take arcs in order of weight.
  - Reject the arc if it would form a cycle.
  - Add it to the tree if it does not form a cycle.
- Continue taking each arc in turn until all vertices are connected.

b **i** GH, GI, HF, FD, DA, AB, AC, DE

ii GH, AB, AC, AD, reject BD, DF, GI, reject BC, FH, reject DG, DE, reject EG, reject HI, reject CE.

c weight is 76

d Prim's algorithm

- It is easily converted into matrix form.
- It is difficult to use Kruskal's algorithm because it is difficult to check for cycles in a large network.

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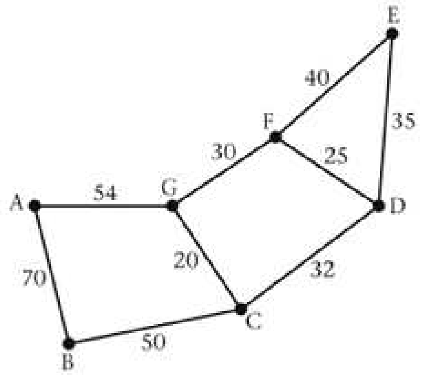
1 Review Exercise
Exercise A, Question 6

Question:



The diagram shows 7 locations A, B, C, D, E, F and G which are to be connected by pipelines. The arcs show the possible routes. The number on each arc gives the cost, in thousands of pounds, of laying that particular section.

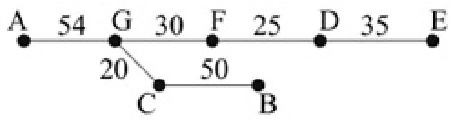
a Use Kruskal's algorithm to obtain a minimum spanning tree for the network, giving the order in which you selected the arcs.

b Draw your minimum spanning tree and find the least cost of pipelines. ***E***

Solution:

a GC, FD, GF, reject CD, ED, reject EF, BC, AG, reject AB.

b



$$\text{cost} = (20+25+30+35+50+54)\times 1000 = £214\,000$$

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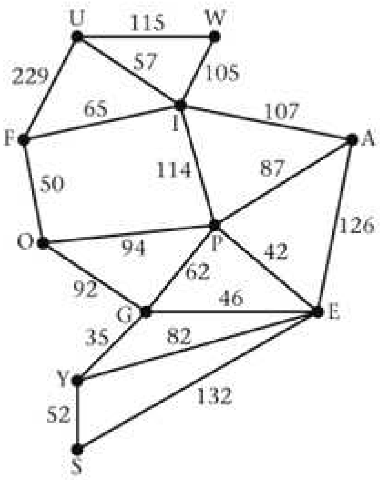
1 Review Exercise
Exercise A, Question 7

Question:



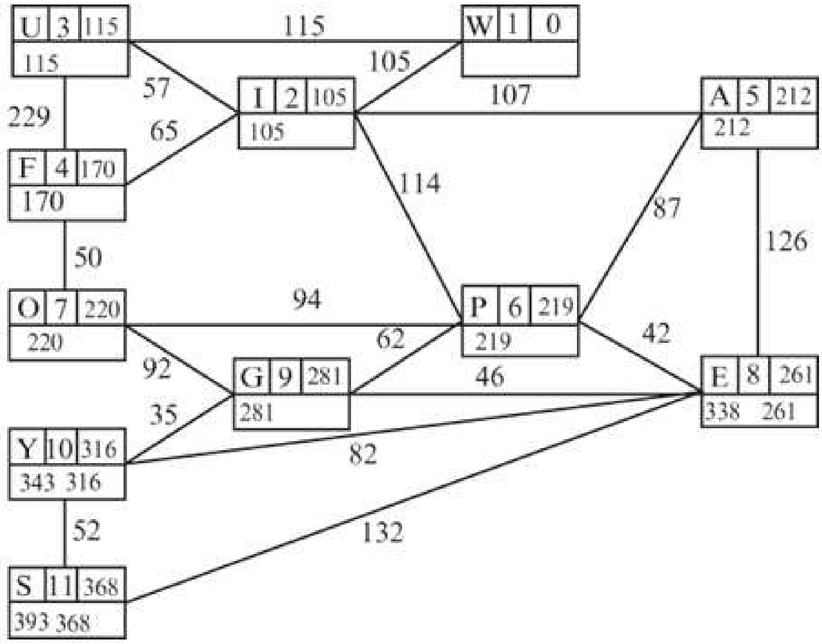
The network above represents the distances, in miles between eleven places A, E, F, G, I, O, P, S, U, W and Y.

a Use Dijkstra's algorithm to find the shortest route from W to S. State clearly
- **i** the order in which you labelled the vertices,
- **ii** how you determined the shortest route from your labelling,
- **iii** the places on the shortest route,
- **iv** the shortest distance.

b Explain how part **a** could have been completed so that the distance from A to S could also have been obtained without further calculation. (You are not required to find this distance.) ***E***

Solution:

a i



ii

$368 - 52 = 316$ SY
$316 - 35 = 281$ GS
$281 - 62 = 219$ PG
$219 - 114 = 105$ IP
$105 - 105 = 0$ WI

iii Shortest route is W–I–P–G–Y–S

iv length 368 miles

b If we started at S, the algorithm would find the least distance from S to each vertex, so in finding S to W we could have found S to A.

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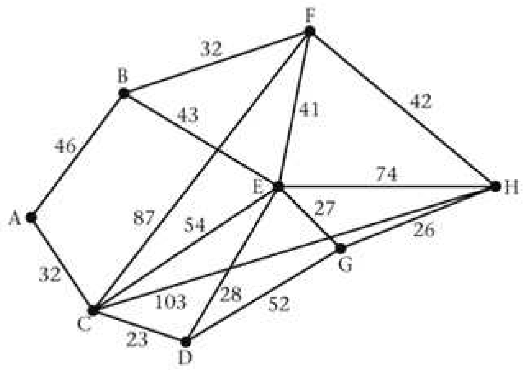
1 Review Exercise
Exercise A, Question 8

Question:

The network shows the possible routes between cities A, B, C, D, E, F, G and H.



The number on each arc gives the cost, in pounds, of taking that part of the route.
Use Dijkstra's algorithm to determine the cheapest route from A to H and its cost.

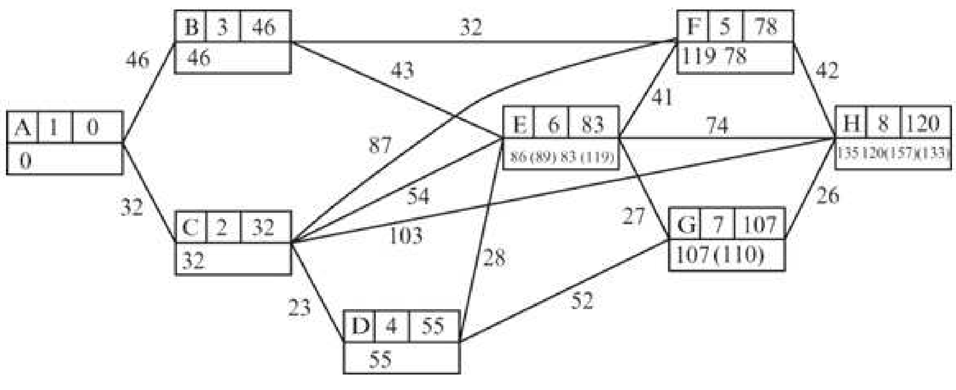
Your solution must indicate clearly how you have applied the algorithm.
State clearly

a the order in which the vertices are labelled,

b how you used your labelled diagram to decide on the cheapest route. *E*

Solution:

a



b $120 - 42 = 78$ FH
$78 - 32 = 46$ BF
$46 - 46 = 0$ AB
route is A – B – F – H cost £120

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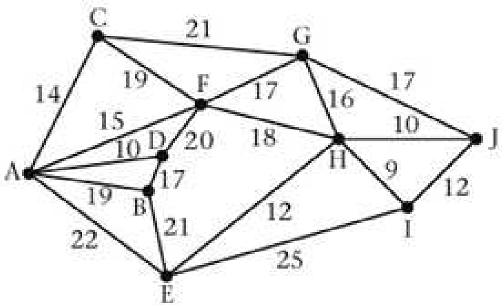
1 Review Exercise
Exercise A, Question 9

Question:



The network above models the roads linking ten towns A, B, C, D, E, F, G, H, I and J. The number on each arc is the journey time in minutes, along the road.

Alice lives in town A and works in town J.

a Use Dijkstra's algorithm to find the quickest route for Alice to travel to work each morning.

State clearly

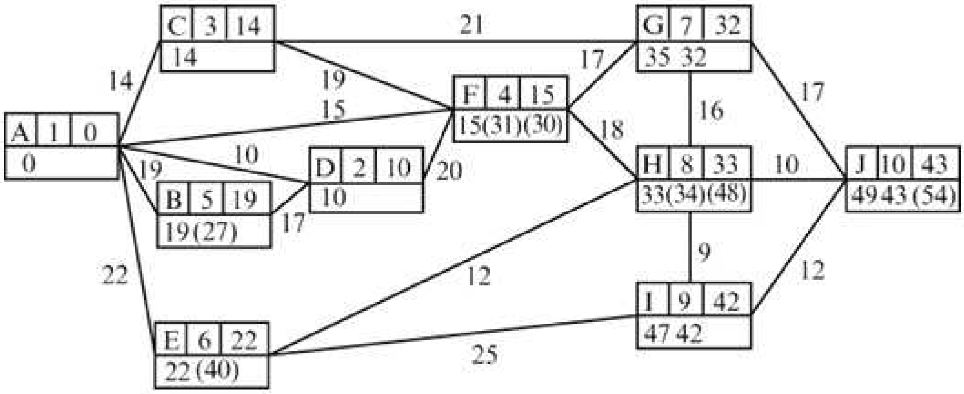
i the order in which all the vertices were labelled,

ii how you determined the quickest route from your labelling.

b On her return journey from work one day Alice wishes to call in at the supermarket located in town C.
Explain briefly how you would find the quickest route in this case. ***E***

Solution:

a **i**



ii $43 - 10 = 33$ HJ

$33 - 18 = 15$ FH

$15 - 15 = 0$ AF

route is A – F – H – J length 43 minutes

b Use the algorithm starting at J to find the shortest route from J to C, then add arc CA.

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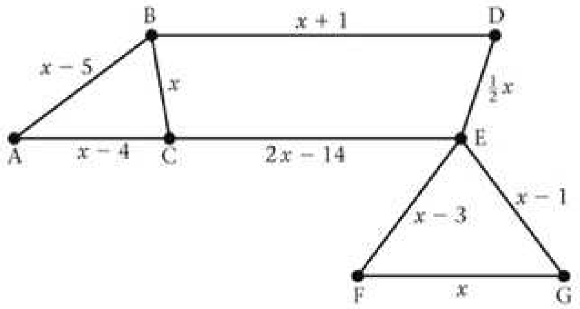
1 Review Exercise
Exercise A, Question 10

Question:

a Explain why it is impossible to draw a network with exactly three odd vertices.



The route inspection problem is solved for the network above and the length of the route is found to be 100.

b Determine the value of x, showing your working clearly. ***E***

Solution:

a Each edge contributes 1 to the order of the vertices at each end. So each edge contributes 2 to the total sum of the orders.
The sum of the orders is therefore an even number.
If there were an odd number of vertices with odd order the sum would be odd. So there must always be an even (or zero) number of vertices of odd order.

b The only vertices of odd order are B and C, we have to repeat the shortest path between B and C.

If $x \geq 9$ the shortest path is BC (direct).

Weight of network $+\text{BC} = 100$

$$(9\frac{1}{2}x - 26) + x = 100 \Rightarrow x = 12$$

If $x < 9$ the shortest path is BAC of length $2x - 9$
$(9\frac{1}{2}x - 26) + 2x - 9 = 100 \Rightarrow x = 11\frac{17}{23} \not< 9$ so inconsistent.

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1 Review Exercise
Exercise A, Question 11

Question:

a

	A	B	C	D	E	F
A	–	10	12	13	20	9
B	10	–	7	15	11	7
C	12	7	–	11	18	3
D	13	15	11	–	27	8
E	20	11	18	27	–	18
F	9	7	3	8	18	–

The table shows the distances, in metres, between six nodes A, B, C, D, E and F of a network.

i Use Prim's algorithm, starting at A, to solve the minimum connector problem for this table of distances. Explain your method and indicate the order in which you selected the edges.
ii Draw your minimum spanning tree and find its total length.
iii State whether your minimum spanning tree is unique. Justify your answer.

b A connected network N has seven vertices.
i State the number of edges in a minimum spanning tree for N.

A minimum spanning tree for a connected network has n edges.
ii State the number of vertices in the network. ***E***

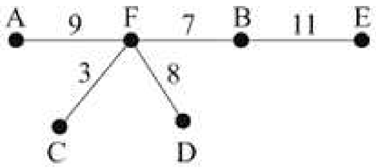
Solution:

a **i** Method:

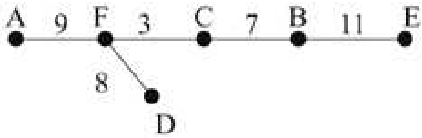
- Start at A and use this to start the tree.
- Choose the shortest edge that connects a vertex already in the tree to a vertex not yet in the tree. Add it to the tree.
- Continue adding edges until all vertices are in the tree.

$$\text{AF, FC}\begin{Bmatrix}\text{FB}\\ \text{or}\\ \text{BC}\end{Bmatrix}\text{, FD, EB}$$

ii



or



iii The tree is not unique, there are 2 of them (see above).

b **i** number of edges $= 7 - 1 = 6$

ii number of vertices $= n + 1$

In a tree the number of edges is always one less than the number of nodes.

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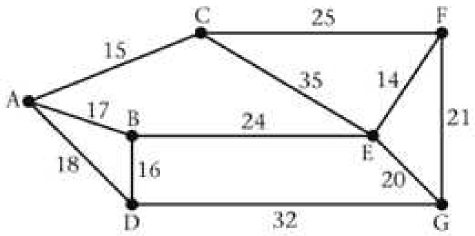
1 Review Exercise
Exercise A, Question 12

Question:

a Describe the differences between Prim's algorithm and Kruskal's algorithm for finding a minimum connector of a network.



b Listing the arcs in the order that you select them, find a minimum connector for the network above, using

i Prim's algorithm,
ii Kruskal's algorithm. ***E***

Solution:

a For example

- In Prim the tree 'grows' in a connected fashion.
- There is no need to check for cycles when using Prim.
- Prim can be adapted to matrix form.
- Prim starts with a vertex, Kruskal with an edge.
- Kruskal must start with the least edge, Prim can start with any verte

b **i** For example AC, AB, BD, EB, EF, EG

ii EF, AC, BD, AB, reject AD, EG, reject FG, BE

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1 Review Exercise
Exercise A, Question 13

Question:

45 56 37 79 46 18 90 81 51

a Using the quick sort algorithm, perform one complete iteration towards sorting these numbers into ascending order.

b Using the bubble sort algorithm, perform one complete pass towards sorting the original list into descending order.

Another list of numbers, in ascending order, is

7 23 31 37 41 44 50 62 71 73 94

c Use the binary search algorithm to locate the number 73 in this list. ***E***

Solution:

a For example, 45 37 18 $\boxed{46}$ 56 79 90 81 51

b For example, 56 45 79 46 37 90 81 51 18

c 1st pivot $\left[\frac{1+11}{2}\right] \rightarrow$ 6th number (44) 73 > 44 so reject 1st to 6th numbers

2nd pivot $\left[\frac{7+11}{2}\right] \rightarrow$ 9th number (71) 73 > 71 so reject 7th to 9th numbers

3rd pivot $\left[\frac{10+11}{2}\right] \rightarrow$ 11th number (94) 73 < 94 so reject 11th number

4th pivot 10th number (73) item found

73 was found as the 10th number in the list.

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1 Review Exercise
Exercise A, Question 14

Question:

The following list gives the names of some students who have represented Britain in the International Mathematics Olympiad.

Roper (R), Palmer (P), Boase (B), Young (Y), Thomas (T), Kenney (K), Morris (M), Halliwell (H), Wicker (W), Garesalingam (G).

a Use the quick sort algorithm to sort the names above into alphabetical order.

b Use the binary search algorithm to locate the name Kenney. ***E***

Solution:

a For example,

R	P	B	Y	T	K	M	H	W	G
B	H	G	$\boxed{K}$	R	P	Y	T	M	W
B	G	$\boxed{H}$		R	P	M	$\boxed{T}$	Y	W
B	$\boxed{G}$			M	$\boxed{P}$	R		$\boxed{W}$	Y
B	G	H	K	M	P	R	T	W	Y

list is in order

b $\left[\frac{10+1}{2}\right] \rightarrow 6$ Palmer Kenney $<$ Palmer reject 6 to 10

list is now

1 Boase
2 Garesalingham
3 Halliwell
4 Kenney
5 Morris

2nd pivot $\left[\frac{1+5}{2}\right] \rightarrow 3.$ Halliwell Kenney $>$ Halliwell reject 1 to 3

list is now
4 Kenney
5 Morris

3rd pivot $\left[\frac{4+5}{2}\right] \rightarrow 5.$ Morris Kenney $<$ Morris reject 5

list is now
4 Kenney
item found

Kenney was found as the 4th name in the list.

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1 Review Exercise
Exercise A, Question 15

Question:

1	Glasgow
2	Newcastle
3	Manchester
4	York
5	Leicester
6	Birmingham
7	Cardiff
8	Exeter
9	Southampton
10	Plymouth

A binary search is to be performed on the names in the list above to locate the name Newcastle.

a Explain why a binary search cannot be performed with the list in its present form.

b Using an appropriate algorithm, alter the list so that a binary search can be performed. State the name of the algorithm you use.

c Use the binary search algorithm on your new list to locate the name Newcastle. ***E***

Solution:

a The list is not in alphabetical order.

b For example, quick sort.

G	N	M	Y	L	B	C	E	S	P
$\boxed{B}$	G	N	M	Y	L	C	E	S	P
	G	C	E	$\boxed{L}$	N	M	Y	S	P
	$\boxed{C}$	G	E		N	M	S	P	$\boxed{Y}$
		$\boxed{E}$	G		N	M	P	$\boxed{S}$	
			G		$\boxed{M}$	N	P		
						N	$\boxed{P}$		
B	C	E	G	L	M	N	P	S	Y

c 1 Birmingham
2 Cardiff
3 Exeter
4 Glasgow
5 Leicester
6 Manchester
7 Newcastle
8 Plymouth
9 Southampton
10 York

$\left[\frac{1+10}{2}\right] \rightarrow 6$ Manchester Newcastle > Manchester so reject 1 to 6

list is now:
7 Newcastle
8 Plymouth
9 Southampton
10 York

$\left[\frac{7+10}{2}\right] \rightarrow 9$ Southampton Newcastle < Southampton so reject 9 to 10

list is now
7 Newcastle
8 Plymouth

$\left[\frac{7+8}{2}\right] \rightarrow 8$ Plymouth Newcastle < Plymouth so reject 8

list is now
7 Newcastle
Final term is 7, Newcastle $\therefore$ name found at 7.

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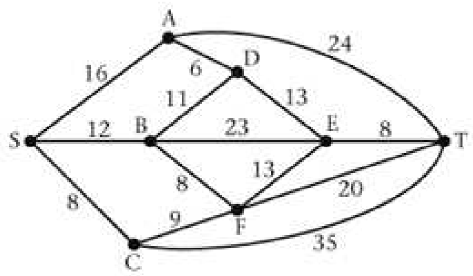
1 Review Exercise
Exercise A, Question 16

Question:



The weighted network shown above models the area in which Bill lives. Each vertex represents a town. The edges represent the roads between the towns. The weights are the lengths, in km, of the roads.

a Use Dijkstra's algorithm to find the shortest route from Bill's home at S to T. Complete all the boxes on the answer sheet and explain clearly how you determined the path of least weight from your labelling.

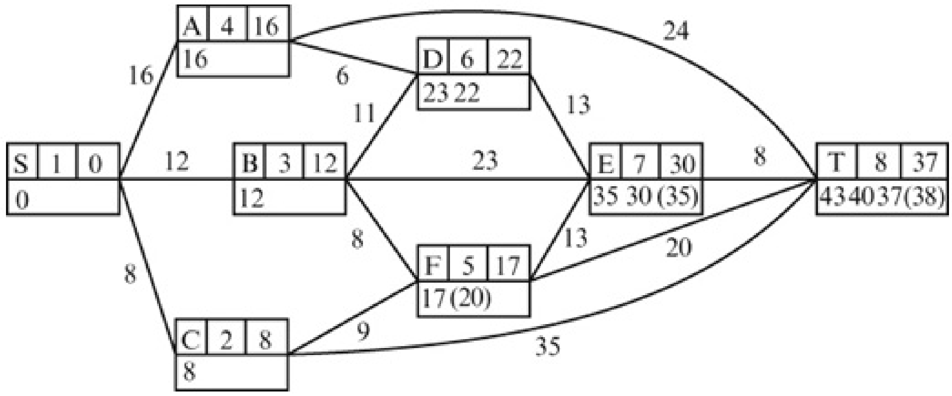
Bill decides that on the way to T he must visit a shop in town E.

b Obtain his shortest route now, giving its length and explaining your method clearly.

E

Solution:

a



For example

$37-20=17$ FT

$17-9=8$ CF

$8-8=0$ SC

shortest path is $S-C-F-T$ length 37

b Need shortest path S to $E+ET$

So $S-C-F-E-T$ length 38

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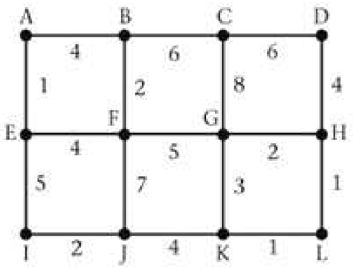
1 Review Exercise
Exercise A, Question 17

Question:



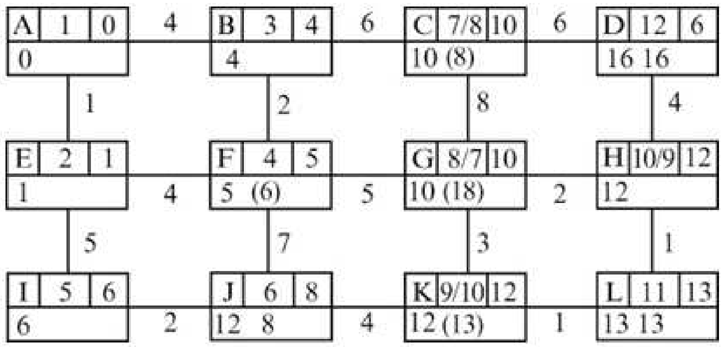
The diagram shows a network of roads. Erica wishes to travel from A to L as quickly as possible. The number on each edge gives the time, in minutes, to travel along the road.

a Use Dijkstra's algorithm to find the quickest route from A to L. Complete all the boxes on the answer sheet and explain clearly how you determined the quickest route from your labelling.

b Show that there is another route which also takes the minimum time. ***E***

Solution:

a



For example

$13-1=12$	HL	or	$13-1=12$	KL
$12-2=10$	GH		$12-4=8$	JK
$10-5=5$	FG		$8-2=6$	IJ
$5-4=1$	EF		$6-5=1$	EI
$1-1=0$	AE		$1-1=0$	AE

Shortest path is $\left\{\begin{matrix} A-E-F-G-H-L \\ A-E-I-J-K-L \end{matrix}\right\}$ length 13

b State the other path given above in part **a**.

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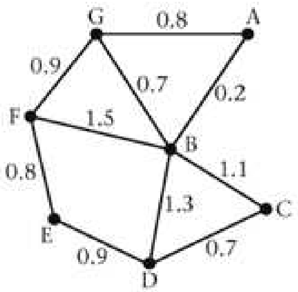
1 Review Exercise
Exercise A, Question 18

Question:



An engineer needs to check the state of a number of roads to see whether they need resurfacing. The roads that need to be checked are represented by the arcs in the diagram. The number on each arc represents the length of that road in km. To check all the roads, he needs to travel along each road at least once. He wishes to minimise the total distance travelled.

The engineer's office is at G, so he starts and ends his journey at G.

a Use an appropriate algorithm to find a route for the engineer to follow. State your route and its length.

The engineer lives at D. He believes he can reduce the distance travelled by starting from home and inspecting all the roads on the way to his office at G.

b State whether the engineer is correct in his belief. If so, calculate how much shorter his new route is. If not, explain why not. ***E***

Solution:

a $BD + FG = 1.3 + 0.9 = 2.2$ ← The odd valencies are at B, D, F, G.

$BF + DG = 1.5 + 2.0 = 3.5$

$BG + DF = 0.7 + 1.7 = 2.4$

Repeat BD and FG

Route for example $G - F - E - D - B - C - D - B - F - G - A - B - G$
(13 letters in route)
length $8.9 + 2.2 = 11.1$ km

b It would now only be necessary to repeat BF of length $1.5 < 2.2$
length $= 8.9 + 1.5 = 10.4$ km, saving 0.7 km.

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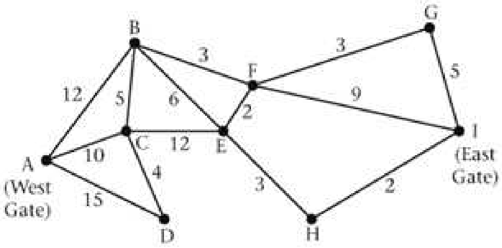
1 Review Exercise
Exercise A, Question 19

Question:



The diagram shows the network of paths in a country park. The number on each path gives its length in km. The vertices A and I represent the two gates in the park and the vertices B, C, D, E, F, G and H represent places of interest.

a Use Dijkstra's algorithm to find the shortest route from A to I. Show all necessary working in the boxes on the answer sheet and state your shortest route and its length.

The park warden wishes to inspect each of the paths to check for frost damage. She has to cycle along each path at least once, starting and finishing at A.

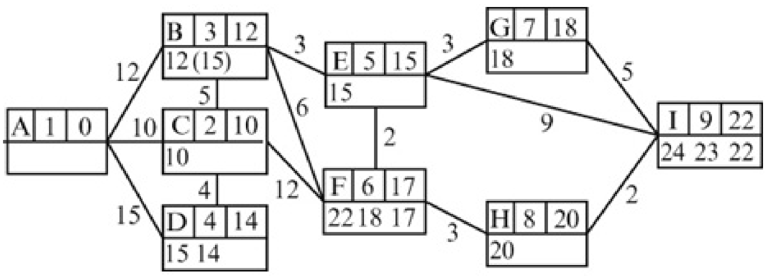
b i Use an appropriate algorithm to find which paths will be covered twice and state these paths.

ii Find a route of minimum length.

iii Find the total length of this shortest route. ***E***

Solution:

a



Shortest route is A – B – E – F – H – I, length 22 km

b **i** Odd vertices are A and I (only), so we need to repeat the shortest route from A to I. This was found in **a**.
so repeat AB, BE, EF, FH, HI.

ii For example A – B – C – A – D – C – E – H – I – H – E – F – I – G – F – E – B – F – B – A (20 letters in route)

iii $91+22=113$ km

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1 Review Exercise
Exercise A, Question 20

Question:

90 50 55 40 20 35 30 25 45

a Use the bubble sort algorithm to sort the list of numbers above into descending order showing the rearranged order after each pass.

Jessica wants to record a number of television programmes onto video tapes. Each tape is 2 hours long. The lengths, in minutes, of the programmes she wishes to record are:

55 45 20 30 30 40 20 90 25 50 35 and 35

b Find the total length of programmes to be recorded and hence determine a lower bound for the number of tapes required.

c Use the first fit decreasing algorithm to fit the programmes onto her 2-hour tapes.

Jessica's friend Amy says she can fit all the programmes onto 4 tapes.

d Show how this is possible. ***E***

Solution:

a For example bubbling left to right

90	50	55	40	20	35	30	25	45
90	55	50	40	35	30	25	45	20
90	55	50	40	35	30	45	25	20
90	55	50	40	35	45	30	25	20
90	55	50	40	45	35	30	25	20
90	55	50	45	40	35	30	25	20

No more changes, list sorted.

b $\frac{475}{120} = 3.96$ so lower bound is 4 tapes

c Bin 1: $90 + 30$
Bin 2: $55 + 50$
Bin 3: $45 + 40 + 35$
Bin 4: $35 + 30 + 25 + 20$
Bin 5: 20

d For example
Bin 1: $90 + 30$
Bin 2: $55 + 35 + 30$
Bin 3: $45 + 40 + 35$
Bin 4: $50 + 25 + 20 + 20$

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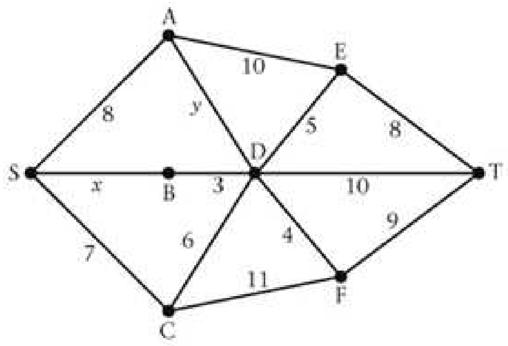
1 Review Exercise

Exercise A, Question 21

Question:



A weighted network is shown above.

Given that the shortest path from S to T is 17 and that $x \geq 0, y \geq 0$:

a **i** explain why A and C cannot lie on the shortest path,

ii find the value of x.

b Given that $x = 12$ and $y \geq 0$, find the possible range of values for the length of the shortest path.

c Give an example of a practical problem that could be solved by drawing a network and finding the shortest path through it. ***E***

Solution:

a **i** Shortest path through A is $18 + y$ or 26 both of which are greater than 17. Shortest path through C is 23, which is greater than 17. So shortest path cannot go through A or C.

ii Shortest path must go through B.

$$S - B - D - T = 13 + x$$
$$13 + x = 17$$
$$x = 4$$

b If $y = 0$ shortest path is $S - A - D - T = 18$

If $y = 5$ shortest path is $S - C - D - T = 23$

so range is 18 to 23.

c For example, a person seeking the quickest route from home to work through a city. The arcs are the roads that may be chosen, the number the time, in minutes, to journey along that road. The nodes represent junctions.

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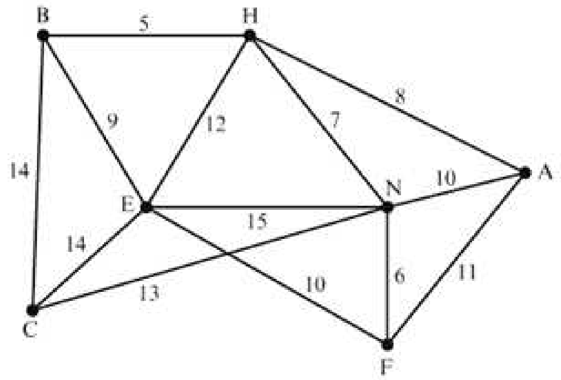
1 Review Exercise
Exercise A, Question 22

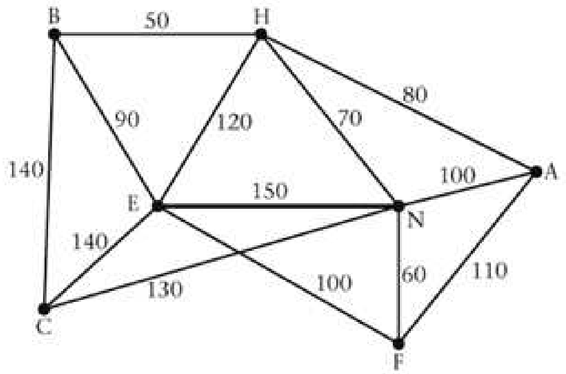
Question:

a Define the terms
 i tree,
 ii spanning tree,
 iii minimum spanning tree.

b State one difference between Kruskal's algorithm and Prim's algorithm, to find a minimum spanning tree.



c Use Kruskal's algorithm to find the minimum spanning tree for the network shown above. State the order in which you included the arcs. Draw the minimum spanning tree and state its length.



This network models a car park. Currently there are two pay-stations, one at E and one at N. These two are linked by a cable as shown. New pay-stations are to be installed at B, H, A, F and C. The number on each arc represents the distance between the pay-stations in metres. All of the pay-stations need to be connected by cables, either directly or indirectly. The current cable between E and N must be included in the final network. The minimum amount of new cable is to be used.

d Using your answer to part **c**, or otherwise, determine the minimum amount of new cable needed. Draw a diagram to show where these cables should be installed. State the minimum amount of new cable needed. ***E***

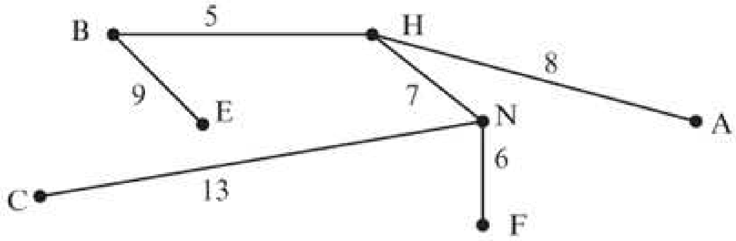
Solution:

a **i** A connected graph with no cycles, loops or multiple edges.
ii A tree that includes all vertices.
iii A spanning tree of minimum total weight.

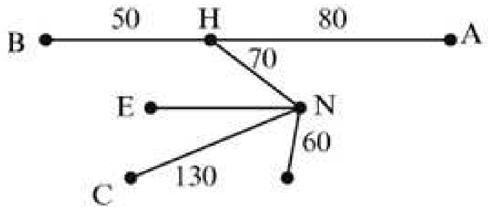
b For example,

- There is no need to check for cycles when using Prim's algorithm.
- Prim's algorithm can start at any vertex, Kruskal's algorithm starts with the shortest arc.
- In Prim's algorithm the tree 'grows' in a connected fashion, with Kruskal's algorithm the tree may not be connected until the end.
- When using Kruskal's algorithm the shortest *arc* is added to the tree (unless it completes a cycle); with Prim's algorithm the nearest unattached *vertex* is added.

c BH, NF, HN, AH, BE, AN, EF, AF, HE, CN, CE, BC, EN
Use BH, NF, HN, AH, BE, reject NA, reject EF, reject AF, reject HE, CN



d



New cable: 390 m

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1 Review Exercise
Exercise A, Question 23

Question:

Nine pieces of wood are required to build a small cabinet. The lengths, in cm, of the pieces of wood are listed below.

20 20 20 35 40 50 60 70 75

Planks, one metre in length, can be purchased at a cost of £3 each.

a The first fit decreasing algorithm is used to determine how many of these planks are to be purchased to make this cabinet. Find the total cost and the amount of wood wasted.

Planks of wood can also be bought in 1.5 m lengths, at a cost of £4 each. The cabinet can be built using a mixture of 1 m and 1.5 m planks.

b Find the minimum cost of making this cabinet. Justify your answer. ***E***

Solution:

a 75 70 60 50 40 35 20 20 20
Bin 1: $75+20$
Bin 2: $70+20$
Bin 3: $60+40$
Bin 4: $50+35$
Bin 5: 20
5 planks needed at a cost of $5\times £3 = £15$
wastage is $5+10+0+15+80=110$ cm

b For example Bin 1 (1.5 m): $75+70$
Bin 2 (1.5 m): $60+50+40$
Bin 3 (1 m): $35+20+20+20$
Cost: $2\times £4+£3=£11$.

or

Bin 1 (1 m): $75+20$
Bin 2 (1.5 m): $70+60+20$
Bin 3 (1.5 m): $50+40+35+20$

1.5 m lengths are better value than 1 m lengths, therefore the solution using as many 1.5 m lengths as possible is preferred.

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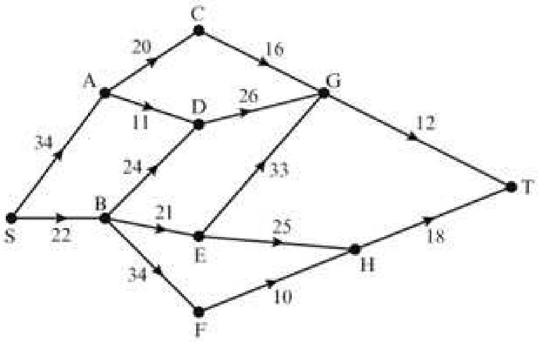
1 Review Exercise
Exercise A, Question 24

Question:



a Use Dijkstra's algorithm to find the shortest route from S to T in this network. Show all necessary working by drawing a diagram. State your shortest route and its length.

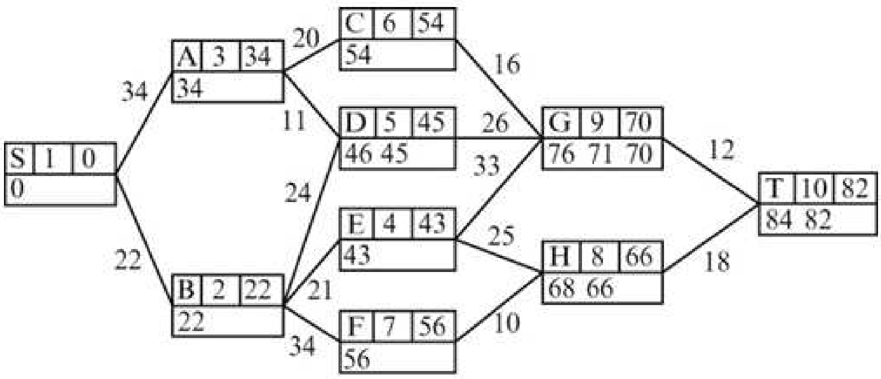
b Explain how you determined the shortest route from your labelling.

c It is now necessary to go from S to T via H. Obtain the shortest route and its length.

E

Solution:

a



Route: S – A – C – G – T length: 82

b For example

$82 - 12 = 70$ GT

$70 - 16 = 54$ CG

$54 - 20 = 34$ AC

$34 - 34 = 0$ SA

c Shortest route from S to H + HT

S – B – F – H – T length: 84

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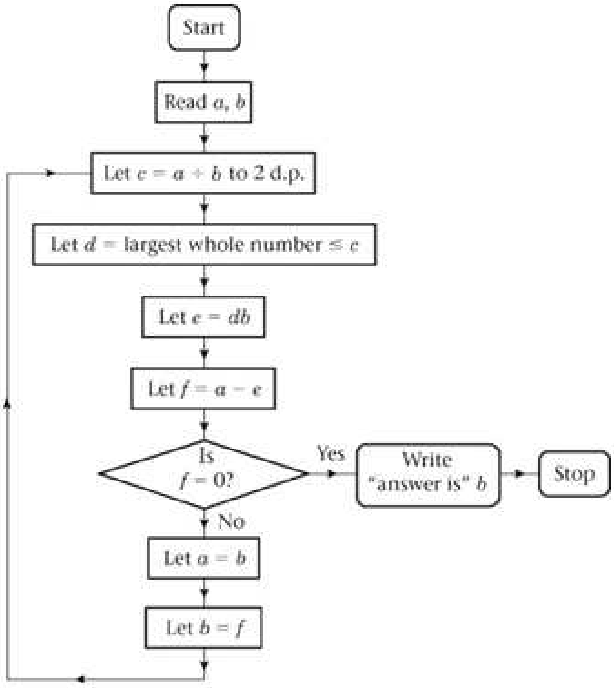
1 Review Exercise
Exercise A, Question 25

Question:

An algorithm is described by the flow chart below.



a Given that $a = 645$ and $b = 255$, draw a table to show the results obtained at each step when the algorithm is applied.

b Explain how your solution to part **a** would be different if you had been given that $a = 255$ and $b = 645$.

c State what the algorithm achieves. ***E***

Solution:

a

a	b	c	d	e	f	$f = 0$?
645	255	2.53	2	510	135	No
255	135	1.89	1	135	120	No
135	120	1.13	1	120	15	No
120	15	8	8	120	0	Yes

answer is 15

b The first row would be

255 645 0.40 0 0 255 No

but the second row would then be the same as the first row in the table above. So in effect this new first line would just be an additional row at the start of the solution.

c Finds the Highest Common Factor of a and b.

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1 Review Exercise
Exercise A, Question 26

Question:

55 80 25 84 25 34 17 75 3 5

a The list of numbers above is to be sorted into descending order. Perform a bubble sort to obtain the sorted list, giving the state of the list after each complete pass.

The numbers in the list represent weights, in grams, of objects which are to be packed into bins that hold up to 100 g.

b Determine the least number of bins needed.

c Use the first-fit decreasing algorithm to fit the objections into bins which hold up to 100 g. ***E***

Solution:

a For example, left to right

55	80	25	84	25	34	17	75	3	5
80	55	84	25	34	25	75	17	5	3
80	84	55	34	25	75	25	17	5	3
84	80	55	34	75	25	25	17	5	3
84	80	55	75	34	25	25	17	5	3
84	80	75	55	34	25	25	17	5	3

No more exchanges, sort complete.

b $403 \div 100 = 4.03$ so 5 bins are needed.

c Bin 1: $84 + 5 + 3$
Bin 2: $80 + 17$
Bin 3: $75 + 25$
Bin 4: $55 + 34$
Bin 5: 25

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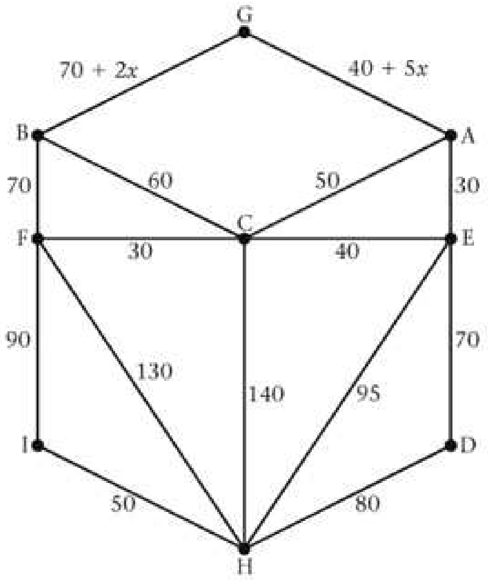
1 Review Exercise
Exercise A, Question 27

Question:



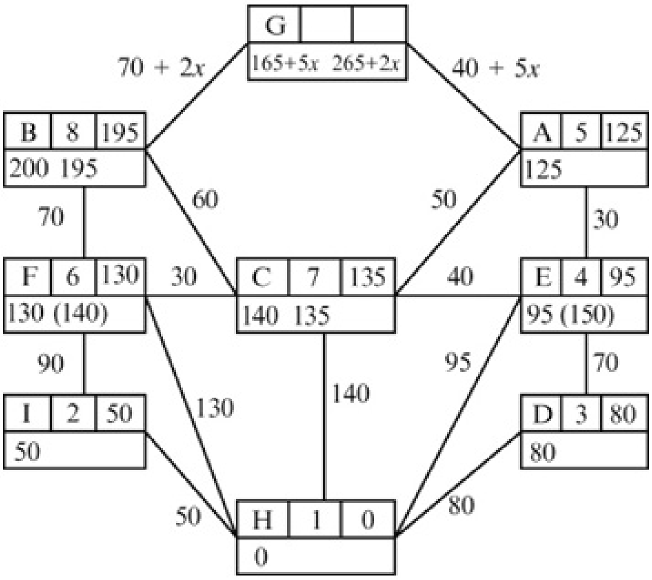
Peter wishes to minimise the time spent driving from his home at H, to a campsite at G. The network above shows a number of towns and the time, in minutes, taken to drive along these roads is expressed in terms of x, where $x \geq 0$.

a Use Dijkstra's algorithm to find two routes from H to G (one via A and one via B) that minimise the travelling time from H to G. State the length of each route in terms of x.

b Find the range of values of x for which Peter should follow the route via A. ***E***

Solution:

a



Via A H − E − A − G length $165+5x$
Via B H − E − C − B − G length $265+2x$

b $165+5x = 265+2x \Rightarrow x = 33\frac{1}{3}$

So range is $0 \le x \le 33\frac{1}{3}$

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1 Review Exercise
Exercise A, Question 28

Question:

	A	B	C	D	E	F
A	–	7	3	–	8	11
B	7	–	4	2	–	7
C	3	4	–	5	9	–
D	–	2	5	–	6	3
E	8	–	9	6	–	–
F	11	7	–	3	–	–

The matrix represents a network of roads between six villages A, B, C, D, E and F. The value in each cell represents the distance, in km, along these roads.

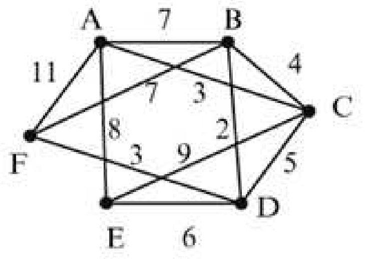
a Show this information on a diagram.

b Use Kruskal's algorithm to determine the minimum spanning tree. State the order in which you include the arcs and the length of the minimum spanning tree.

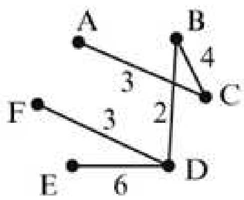
c Starting at D, use Prim's algorithm on the matrix given to find the minimum spanning tree. State the order in which you include the arcs. ***E***

Solution:

a



b BD, $\left\{\begin{matrix}\text{AC}\\\text{DF}\end{matrix}\right\}$, BC, reject CD, DE



c DB, DF, BC, CA, DE
length of tree 18 km

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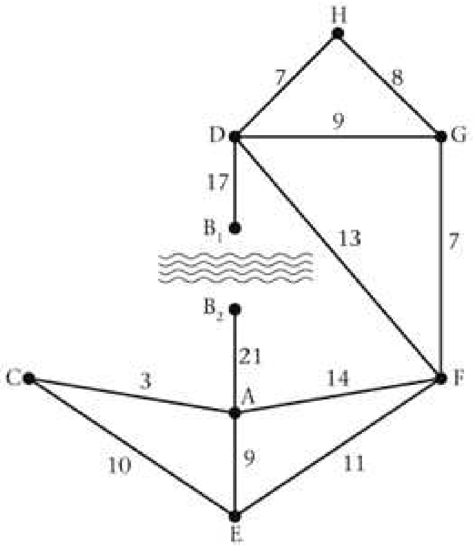
1 Review Exercise
Exercise A, Question 29

Question:



The diagram shows a network of roads connecting villages. The length of each road, in km, is shown. Village B has only a small footbridge over the river which runs through the village. It can be accessed by two roads, from A and D.

The driver of a snowplough, based at F, is planning a route to enable her to clear all the roads of snow. The route should be of minimum length. Each road can be cleared by driving along it once. The snowplough cannot cross the footbridge.

Showing all your working and using an appropriate algorithm,

a find the route the driver should follow, starting and ending at F, to clear all the roads of snow. Give the length of this route.

The local authority decides to build a road bridge over the river at B. The snowplough will be able to cross the road bridge.

b Reapply the algorithm to find the minimum distance the snowplough will have to travel (ignore the length of the new bridge). ***E***

Solution:

a Odd vertices are B_1, B_2, E, G

$B_1B_2 + EG = 65 + 18 = 83$

$B_1E + B_2G = 41 + 42 = 83$

$B_1G + B_2E = 26 + 30 = 56$

Repeat $B_1D, DG, \quad B_2A, AE$

Route: For example,

$F - A - E - A - B_2 - A - C - E - F - G - D - H - G - D - B_1 - D - F$

(All correct routes have 17 letters in their 'word')

length $= 129 + 56 = 185$ km

b Now only the route between E and G needs repeating

so repeat $EF + FG = 18$

length of new route $= 129 + 18$

$= 147$ km

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1 Review Exercise
Exercise A, Question 30

Question:

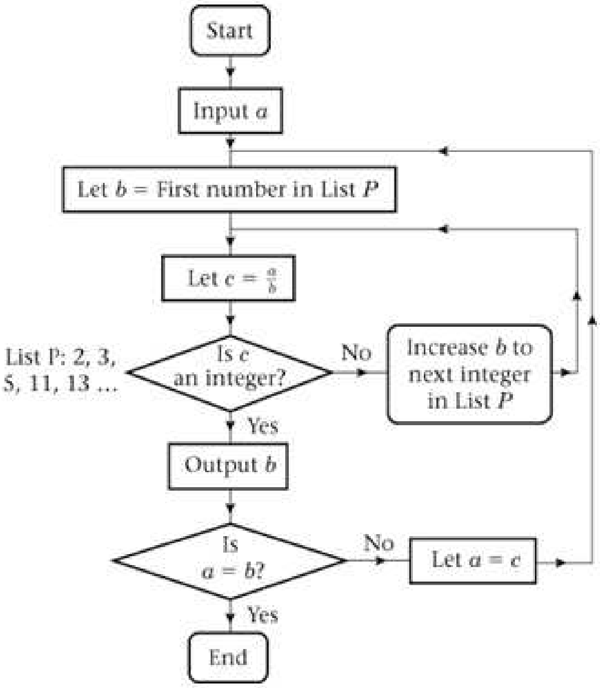
This diagram describes an algorithm in the form of a flow chart, where a is a positive integer.

List P, which is referred to in the flow chart, comprise the prime numbers 2, 3, 5, 7, 11, 13, 17, ...

a Starting with $a = 90$, implement this algorithm. Show your working in the table.

b Explain the significance of the output list.

c Write down the final value of c for any initial value of a.



E

Solution:

a

a	b	c	Integer?	Output list	$a = b$?
90	2	45	Y	2	N
45	2	22.5	N		
45	3	15	Y	3	N
15	2	7.5	N		
15	3	5	Y	3	N
5	2	2.5	N		
5	3	$1\frac{2}{3}$	N		
5	5	1	Y	5	Y

Output list: 2, 3, 3, 5

b Expresses a as a product of prime factors

c $c = 1$ (since we stop when $a = b$ and $c = \frac{a}{b}$)

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1 Review Exercise
Exercise A, Question 31

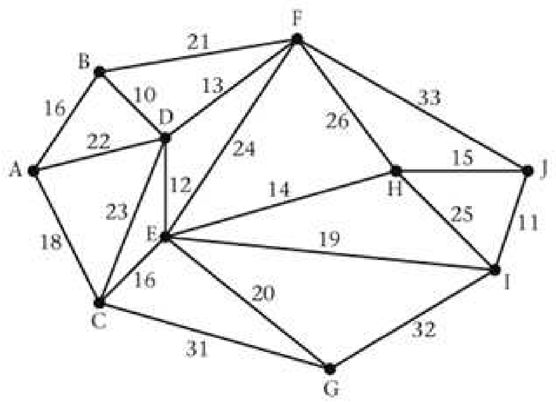
Question:

The network opposite represents the journey time, in minutes, between ten Midland towns.

a Use Dijkstra's algorithm to find the quickest route between A and J. Your solution must indicate clearly how you applied the algorithm, including

i the order in which the vertices were labelled,

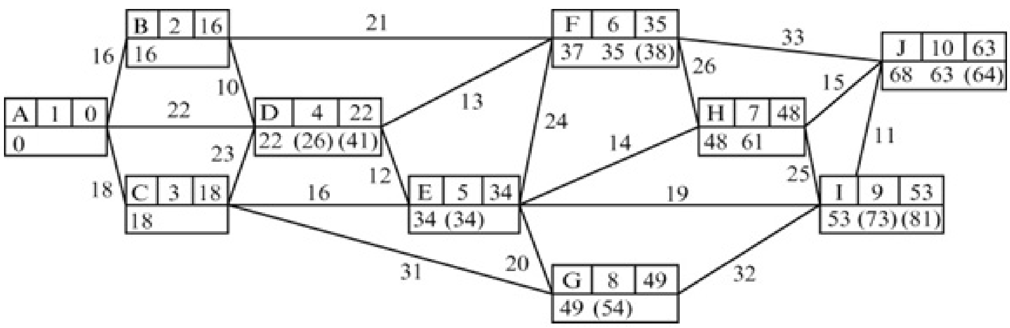
ii how you determined your quickest route from your labelling.



b Is the route you have found the only quickest route? Give a reason for your answer. ***E***

Solution:

a i



ii For example

$63-15=48$	HJ		$63-15=48$	HJ
$48-14=34$	EH	*or*	$48-14=34$	EH
$34-16=18$	CE		$34-12=22$	DE
$18-18=0$	AC		$22-22=0$	AD

Route A – C – E – H – J or A – D – E – H – J length 63

b No, it is not the only route, there are two shortest routes (see above).

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Review Exercise 2
Exercise A, Question 1

Question:

Define the terms
a bipartite graph,
b alternating path,
c matching,
d complete matching. ***E***

Solution:

a A graph consisting of
- two distinct sets of vertices X and Y in which
- arcs can only join a vertex in X to a vertex in Y.

b
- A path from an unmatched vertex in X to an unmatched vertex in Y
- which alternately uses arcs not in/in the matching.

Two points to make in each of these responses.

c The one-to-one pairing of some elements of X with elements of Y.

d A one-to-one matching between all elements of X and Y.

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Review Exercise 2
Exercise A, Question 2

Question:

The organiser of a sponsored walk wishes to allocate each of six volunteers, Alan, Geoff, Laura, Nicola, Philip and Sam to one of the checkpoints along the route. Two volunteers are needed at checkpoint 1 (the start) and one volunteer at each of checkpoints 2, 3, 4 and 5 (the finish). Each volunteer will be assigned to just one checkpoint. The table shows the checkpoints each volunteer is prepared to supervise.

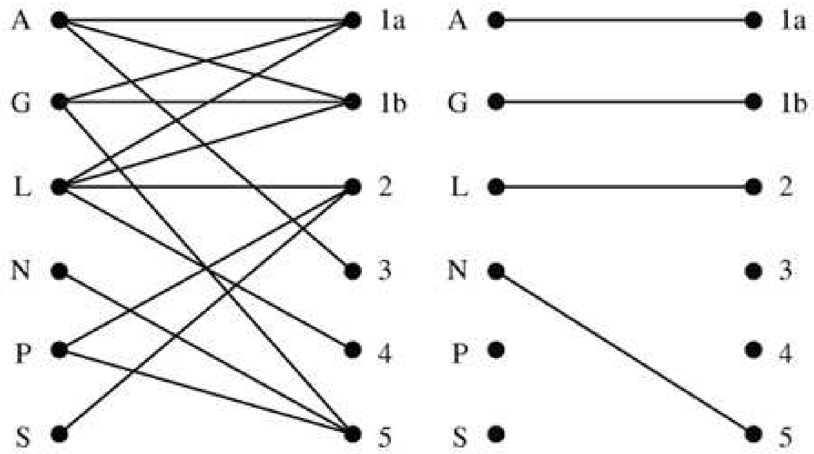
Name	Checkpoints
Alan	1 or 3
Geoff	1 or 5
Laura	2, 1 or 4
Nicola	5
Philip	2 or 5
Sam	2

Initially Alan, Geoff, Laura and Nicola are assigned to the first checkpoint in their individual list.

a Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.
b Starting from this initial matching, use the maximum matching algorithm to find an improved matching. Clearly list any alternating paths you use.
c Explain why it is not possible to find a complete matching. ***E***

Solution:

a



b Six possible alternating paths – only one needed

i $P-2=L-4$ change status $P=2-L=4$
ii $P-2=L-1=G-4$ change status $P=2-L=1-G=4$
iii $P-2=L-1=A-3$ change status $P=2-L=1-A=3$
iv $S-2=L-4$ change status $S=2-L=4$
v $S-2=L-1=G-4$ change status $S=2-L=1-G=4$
vi $S-2=L-1=A-3$ change status $S=2-L=1-A=3$

Giving matchings as follows

Giving matchings as follows

Person	(i)	(ii)	(iii)	(iv)	(v)	(vi)
A	1	1	3	1	1	3
G	1	4	1	1	4	1
L	4	1	1	4	1	1
N	5	5	5	5	5	5
P	2	2	2			
S				2	2	2

c For example, N must do 5 and S must do 2. This leaves P without a task.

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Review Exercise 2
Exercise A, Question 3

Question:

Ann, Bryn, Daljit, Gareth and Nickos have all joined a new committee. Each of them is to be allocated to one of five jobs 1, 2, 3, 4 or 5. The table shows each member's preferences for the jobs.

Ann	1 or 2
Bryn	3 or 1
Daljit	2 or 4
Gareth	5 or 3
Nickos	1 or 2

Initially Ann, Bryn, Daljit and Gareth are allocated the first job in their lists shown in the table.

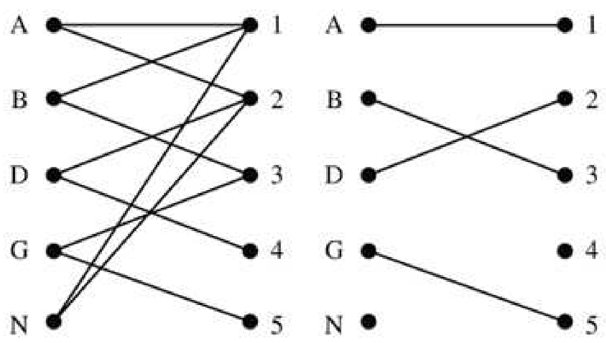
a Draw a bipartite graph to model the preferences shown in the table and indicate, in a distinctive way, the initial allocation of jobs.

b Use the matching improvement algorithm to find a complete matching, showing clearly your alternating path.

c Find a second alternating path from the initial allocation. ***E***

Solution:

a



b Possible paths

N — $1 = A - 2 = D - 4$

N — $2 = D - 4$

Breakthrough.
Choose one of these.

Change status

$N = 1 - A = 2 - D = 4$ giving matching $A = 2$ $B = 3$ $D = 4$ $G = 5$ $N = 1$

or

$N = 2 - D = 4$ giving matching $A = 1$ $B = 3$ $D = 4$ $G = 5$ $N = 2$

c Give the other alternating path.

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Review Exercise 2
Exercise A, Question 4

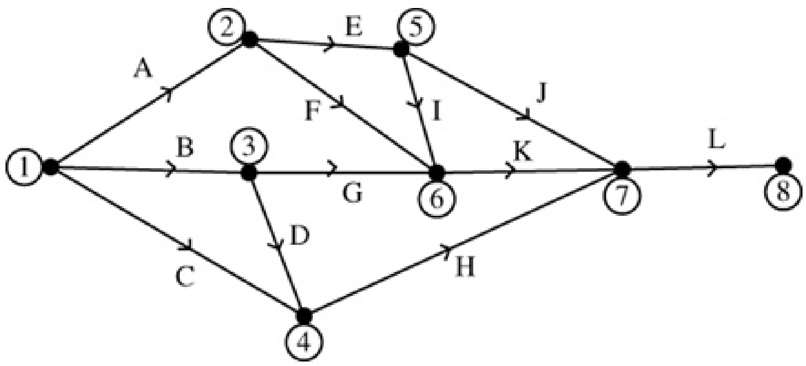
Question:

The precedence table for activities involved in a small project is shown below.

Activity	Preceding activities
A	–
B	–
C	–
D	B
E	A
F	A
G	B
H	C, D
I	E
J	E
K	F, G, I
L	H, J, K

Draw an activity network, using activity on edge and without using dummies, to model this project. ***E***

Solution:



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Review Exercise 2
Exercise A, Question 5

Question:

The precedence table for activities involved in manufacturing a toy is shown below.

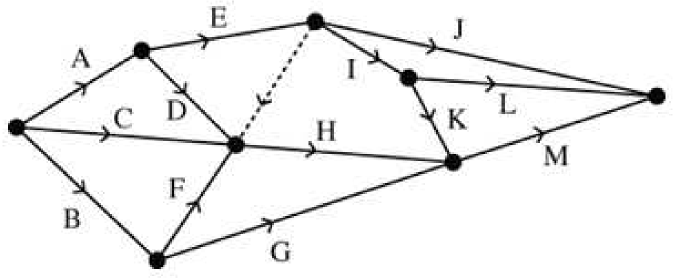
Activity	Preceding activities
A	–
B	–
C	–
D	A
E	A
F	B
G	B
H	C, D, E, F
I	E
J	E
K	I
L	I
M	G, H, K

a Draw an activity network, using activity on arc, and exactly one dummy, to model the manufacturing process.

b Explain briefly why it is necessary to use a dummy in this case. ***E***

Solution:

a



b For example I and J depend only on E.
H depends on C, D, E and F.

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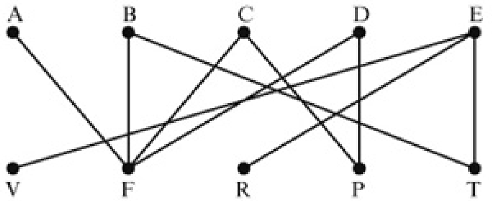
Review Exercise 2
Exercise A, Question 6

Question:



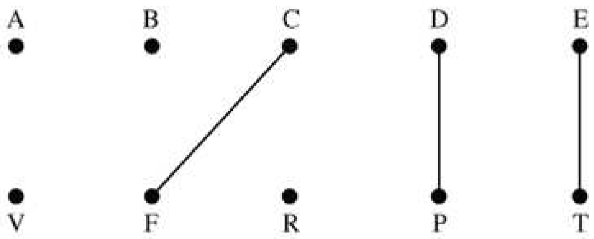
The five winners of a competition are Mr Adams (A), Mr Brown (B), Miss Church (C), Mrs Drain (D) and Ms Eagle (E). The prizes are five cars; a Vauxhall (V), a Ford (F), a Rover (R), a Peugeot (P) and a Toyota (T). The winners' preferences are summarised in the bipartite graph G shown above.

The organiser of the competition matches C with F, D with P and E with T.

a Indicate clearly this matching M on a matching diagram.
b Find an alternating path for M in G, starting at B. Use this to construct an improved matching M'. Show M' on another diagram.
c Show that there is no alternating path for M' in G. ***E***

Solution:

a



b Two possible paths – choose only one.
i $B - T = E - V$ change status $B = T - E = V$
or
ii $B - T = E - R$ change status $B = T - E = R$

Improved matching
i $A = ?$ $B = T$ $C = F$ $D = P$ $E = V$
or
ii $A = ?$ $B = T$ $C = F$ $D = P$ $E = R$

c For example, E is the only person who prefers V and the only person prefers R.
So a complete matching is not possible, so there cannot be a second alternating path.
or

A — $F = C - P = D - F$ loop
A — $P = D - F = C - P$ loop

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Review Exercise 2
Exercise A, Question 7

Question:

At Tesafe supermarket there are 5 trainee staff, Homan (H), Jenna (J), Mary (M), Tim (T) and Yoshie (Y). They each must spend one week in each of 5 departments, Delicatessen (D), Frozen foods (F), Groceries (G), Pet foods (P), Soft drinks (S). Next week every department requires exactly one trainee. The table below shows the departments in which the trainees have yet to spend time.

Trainee	Departments
H	D, F, P
J	G, D, F
M	S, P, G
T	F, S, G
Y	D

Initially H, J, M and T are allocated to the first department in their list.

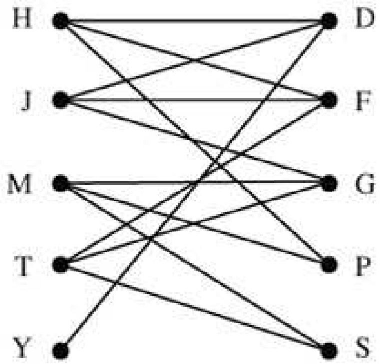
a Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.

Starting from this matching,

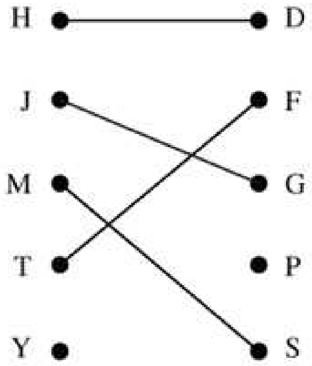
b use the maximum matching algorithm to find a complete matching. You must make clear your alternating path and your complete matching. ***E***

Solution:

a



b



Y – D = H – P breakthrough

Y – D = H – F = T – S = M – P breakthrough

Change status either

i Y = D – H = P giving matching H = P J = G M = S T = F Y = D

or

ii Y = D – H = F – T = S – M = P giving matching
H = F J = G M = P T = S Y = D

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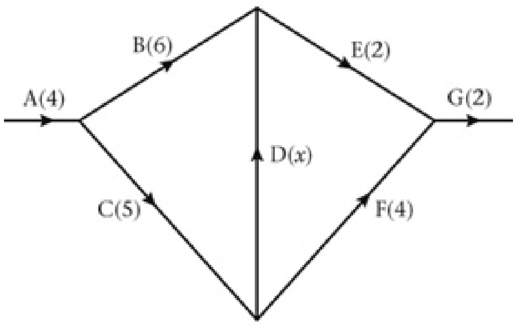
Review Exercise 2
Exercise A, Question 8

Question:

a Draw an activity network for the project described in this precedence table, using as few dummies as possible.

Activity	Must be preceded by:
A	–
B	A
C	A
D	A
E	C
F	C
G	B, D, E, F
H	B, D, E, F
I	F, D
J	G, H, I
K	F, D
L	K

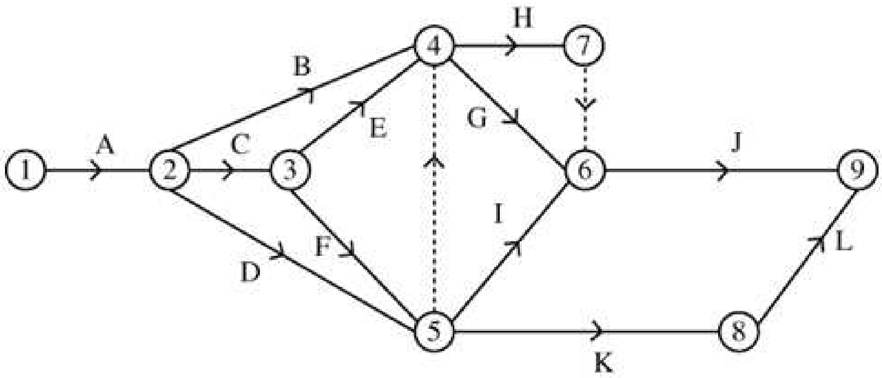
b A different project is represented by the activity network shown below. The duration of each activity is shown in brackets.



Find the range of values of x that will make D a critical activity. ***E***

Solution:

a



b D will only be critical if it lies on the longest path

Path A to G	Length
A – B – E – G	14
A – C – F – G	15
A – C – D – E – G	$13+x$

So we need $13+x$ to be the longest, or equal longest

$13+x \geq 15$

$x \geq 2$

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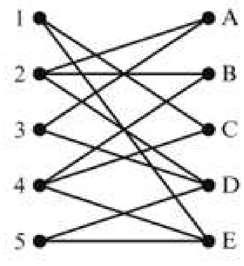
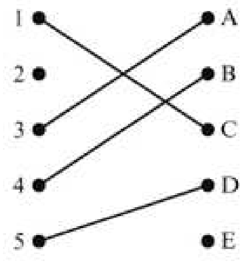
Review Exercise 2
Exercise A, Question 9

Question:



a



b

Five members of staff 1, 2, 3, 4 and 5 are to be matched to five jobs A, B, C, D and E. A bipartite graph showing the possible matchings is given in **a** and an initial matching M is given in **b**.

There are several distinct alternating paths that can be generated from M. Two such paths are
$2-B=4-E$ and
$2-A=3-D=5-E$.

a Use each of these two alternating paths, in turn, to write down the complete matchings they generate.

Use the maximum matching algorithm and the initial matching M,

b find two further distinct alternating paths, making your reasoning clear. ***E***

Solution:

a $2-B=4-E$ change status $2=B-4=E$
giving matching $1=C \quad 2=B \quad 3=A \quad 4=E \quad 5=D$
$2-A=3-D=5-E$ change status $2=A-3=D-5=E$
giving matching $1=C \quad 2=A \quad 3=D \quad 4=B \quad 5=E$

b

$A=3-D=5-E$ (given)
$C=1-E$ breakthrough
2 —— $B=4$
E (given)
$D=5$
E breakthrough

so the two further paths are: $2-B=4-C=1-E$
and $2-D=5-E$

Solutionbank D1
Edexcel AS and A Level Modular Mathematics

Review Exercise 2
Exercise A, Question 10

Question:

Two fertilisers are available, a liquid X and a powder Y. A bottle of X contains 5 units of chemical A, 2 units of chemical B and $\frac{1}{2}$ unit of chemical C. A packet of Y contains 1 unit of A, 2 units of B and 2 units of C.
A professional gardener makes her own fertiliser. She requires at least 10 units of A, at least 12 units of B and at least 6 units of C.
She buys x bottles of X and y packets of Y.

a Write down the inequalities which model this situation.

b On the grid provided construct and label the feasible region.

A bottle of X costs £2 and a packet of Y costs £3.

c Write down an expression, in terms of x and y, for the total cost £T.

d Using your graph, obtain the values of x and y that give the minimum value of T. Make your method clear and calculate the minimum value of T.

e Suggest how the situation might be changed so that it could no longer be represented graphically. ***E***

Solution:

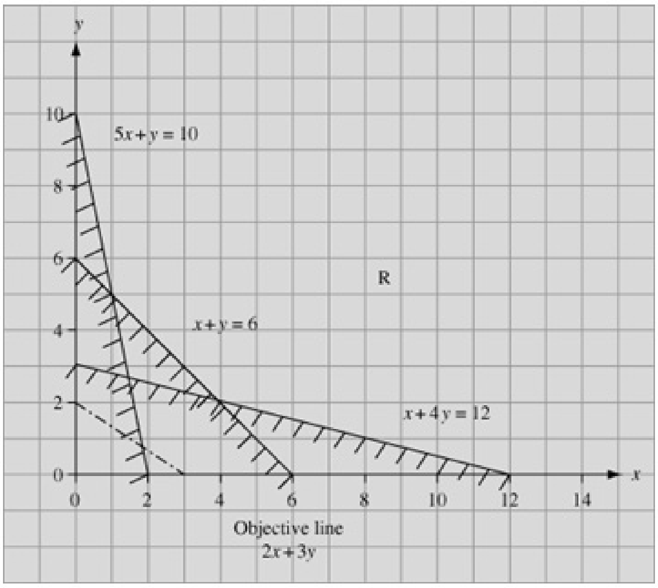
a Chemical A $5x+y\geq 10$

Chemical B $2x+2y\geq 12$ $[x+y\geq 6]$

Chemical C $\frac{1}{2}x+2y\geq 6$ $[x+4y\geq 12]$

$x, y\geq 0$

b



c $T=2x+3y$

d $(4, 2)$ $T=14$

e If there were 3 or more variables the problem could not be solved graphically.
So adding a third fertiliser Z, would mean a graphical method could not be used.

Solutionbank D1

Edexcel AS and A Level Modular Mathematics

Review Exercise 2
Exercise A, Question 11

Question:

At a water sports centre there are five new instructors, Ali (A), George (G), Jo (J), Lydia (L) and Nadia (N). They are to be matched to five sports, canoeing (C), scuba diving (D), surfing (F), sailing (S) and water skiing (W).

The table indicates the sports each new instructor is qualified to teach.

Instructor	Sport
A	C, F, W
G	F
J	D, C, S
L	S, W
N	D, F

Initially, A, G, J and L are each matched to the first sport in their individual list.

a Draw a bipartite graph to model this situation and indicate the initial matching in a distinctive way.

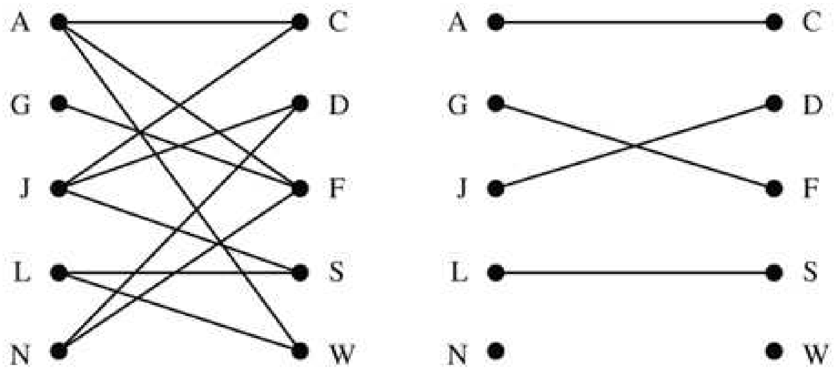
b Starting from this initial matching, use the maximum matching algorithm to find a complete matching. You must clearly list any alternating paths used.

Given that on a particular day J must be matched to D,

c explain why it is no longer possible to find a complete matching. ***E***

Solution:

a



b

N – D = J – C = A – W

N – D = J – S = L – W

N – F = G (stop)

Breakthrough, so choose just one of these.

Change status to give

N = D – J = C – A = W giving matching A = W G = F J = C L = S N = D
N = D – J = S – L = W giving matching A = C G = F J = S L = W N = D

c If J does D, N must do F leaving G without a task.

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Review Exercise 2
Exercise A, Question 12

Question:

A company produces two types of self-assembly wooden bedroom suites, the 'Oxford' and the 'York'. After the pieces of wood have been cut and finished, all the materials have to be packaged. The table below shows the time, in hours, needed to complete each stage of the process and the profit made, in pounds, on each type of suite.

	Oxford	**York**
Cutting	4	6
Finishing	3.5	4
Packaging	2	4
Profit (£)	300	500

The times available each week for cutting, finishing and packaging are 66, 56 and 40 hours respectively.
The company wishes to maximise its profit.
Let x be the number of Oxford, and y be the number of York suites made each week.

a Write down the objective function.

b In addition to

$$2x+3y \le 33,$$
$$x \ge 0,$$
$$y \ge 0.$$

find two further inequalities to model the company's situation.

c Illustrate all the inequalities, indicating clearly the feasible region.

d Explain how you would locate the optimal point.

e Determine the number of Oxford and York suites that should be made each week and the maximum profit gained.

It is noticed that when the optimal solution is adopted, the time needed for one of the three stages of the process is less than that available.

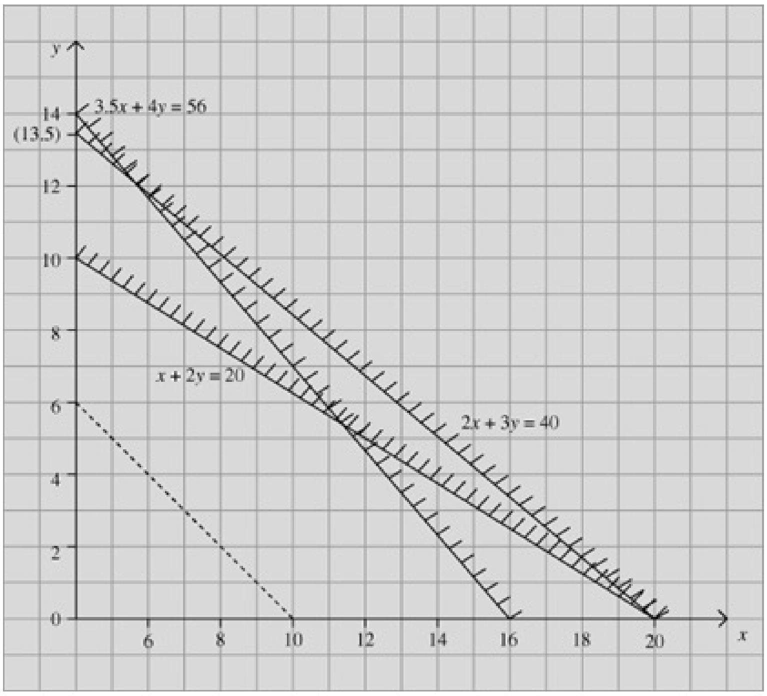
f Identify this stage and state by how many hours the time may be reduced. ***E***

Solution:

a Maximise $P = 300x + 500y$

b Finishing $3.5x + 4y \leq 56 \Rightarrow 7x + 8y \leq 112$ (o.e.)
Packing $2x + 4y \leq 40 \Rightarrow x + 2y \leq 20$ (o.e.)

c



d For example, *point testing*
- Test all corner points in feasible region.
- Find profit at each and select point yielding maximum.

profit line
- Draw profit lines.
- Select point on profit line furthest from the origin.

e Using a correct, complete method.
make 6 Oxford and 7 York profit = £5 300

$(6, 7) \to 5300 \; (14.4, 1.4) \xrightarrow{\text{integer}} (14, 1) \to 4700 \quad (16, 0) \to 4800$

$(0, 10) \to 5000$

f The line $3.5x + 4y = 49$ passes through (6, 7) so reduce *finishing* by 7 hours.

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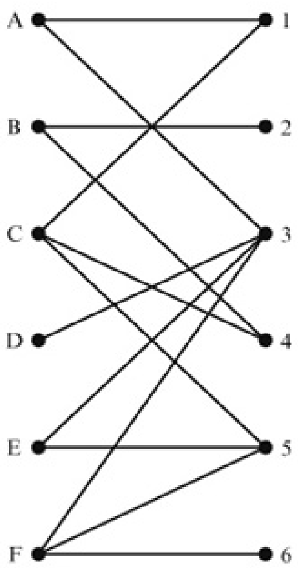
Review Exercise 2
Exercise A, Question 13

Question:



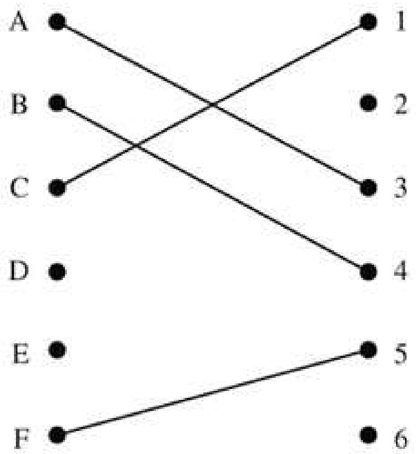
The bipartite graph above shows the possible allocations of people A, B, C, D, E and F to tasks 1, 2, 3, 4, 5 and 6.

An initial matching is obtained by matching the following pairs
A to 3, B to 4, C to 1, F to 5.

a Show this matching in a distinctive way on a diagram.
b Use an appropriate algorithm to find a maximal matching. You should state any alternating paths you have used. ***E***

Solution:

a



b There are six possible pairs of alternating paths – you must only choose one pair.

i D − 3 = A − 1 = C − 4 = B − 2 then E − 5 = F − 6
ii D − 3 = A − 1 = C − 5 = F − 6 then E − 5 = C − 4 = B − 2
iii E − 3 = A − 1 = C − 4 = B − 2 then D − 3 = E − 5 = F − 6
iv E − 3 = A − 1 = C − 5 = F − 6 then D − 3 = E − 5 = C − 4 = B − 2
v E − 5 = F − 6 then D − 3 = A − 1 = C − 4 = B − 2
vi E − 5 = F − 3 = A − 1 = C − 4 = B − 2 then D − 3 = F − 6

changing status, each of these give the same complete matching
A = 1 C = 4 E = 5
B = 2 D = 3 F = 6

Remember to update your 'initial' matching after the first pass through the algorithm. The first alternating path switches things around and your second path needs to take these changes into account.

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Review Exercise 2
Exercise A, Question 14

Question:

Six newspaper reporters Asif (A), Becky (B), Chris (C), David (D), Emma (E) and Fred (F), are to be assigned to six news stories Business (1), Crime (2), Financial (3), Foreign (4), Local (5) and Sport (6). The table shows possible allocations of reporters to news stories. For example, Chris can be assigned to any one of stories 1, 2 or 4.

	1	2	3	4	5	6
A					√	
B	√			√		
C	√	√		√		
D					√	
E			√		√	√
F				√		

a Show these possible allocations on bipartite graph.

A possible matching is
A to 5, C to 1, E to 6, F to 4.

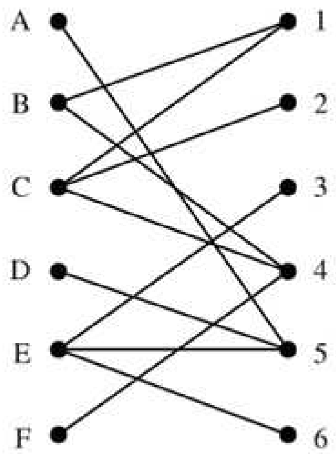
b Show this information, in a distinctive way, on a diagram.

c Use an appropriate algorithm to find a maximal matching. You should list any alternating paths you have used.

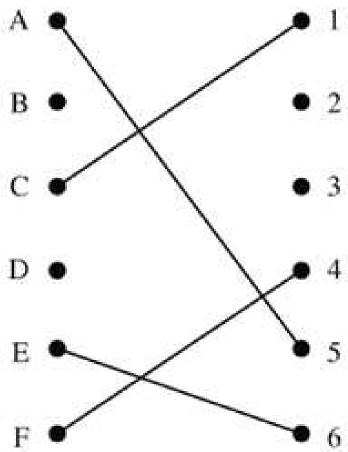
d Explain why it is not possible to find a complete matching. ***E***

Solution:

a



b



c $B-1=C-2$ change status $B=1-C=2$
giving improved matching $A=5$ $B=1$ $C=2$ $D=?$ $E=6$ $F=4$

d For example, E is the only person who can do 3 and also the only person who can do 6, so a 1–1 complete matching is not possible.

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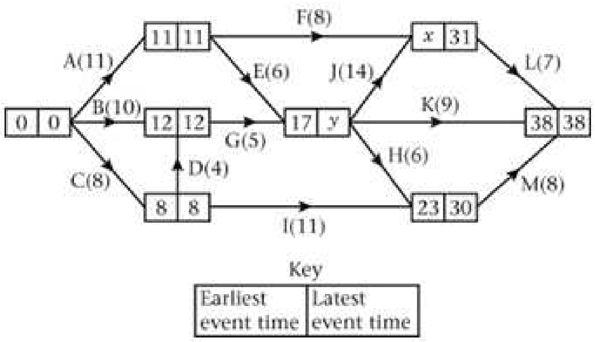
Review Exercise 2
Exercise A, Question 15

Question:



A project is modelled by the activity network in the diagram. The activities are represented by the arcs. One worker is required for each activity. The number in brackets on each arc gives the time, in hours, to complete the activity. The earliest event time and the latest event time are given by the numbers in the left box and right box respectively.

a State the value of x and the value of y.

b List the critical activities.

c Explain why at least 3 workers will be needed to complete this project in 38 hours.

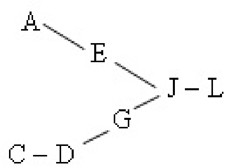
d Schedule the activities so that the project is completed in 38 hours using just 3 workers. You must make clear the start time and finish time of each activity. ***E***

Solution:

$x = 31$ highest of $(11+8)$ and $(17+14)$

a $y = 17$ lowest of $(31-14), (38-9)$ and $(30-6)$

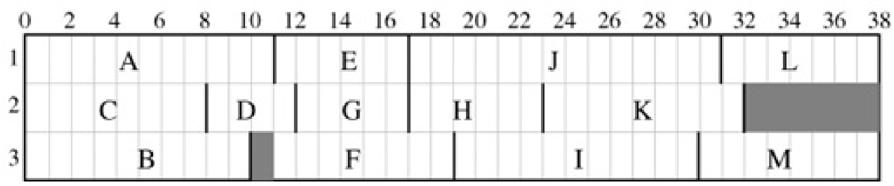
b



All critical activities have a zero total float.

c $107 \div 38 = 2.8$ (1 d.p.) so at least 3 workers needed

d For example,



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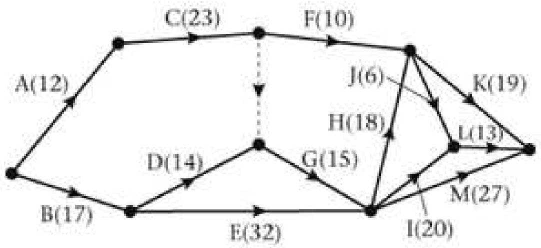
Review Exercise 2
Exercise A, Question 16

Question:

The network shows the activities involved in the process of producing a perfume. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity.



a Calculate the early time and the late time for each event, showing them on a diagram.

b Hence determine the critical activities.

c Calculate the total float time for D.

Each activity requires only one person.

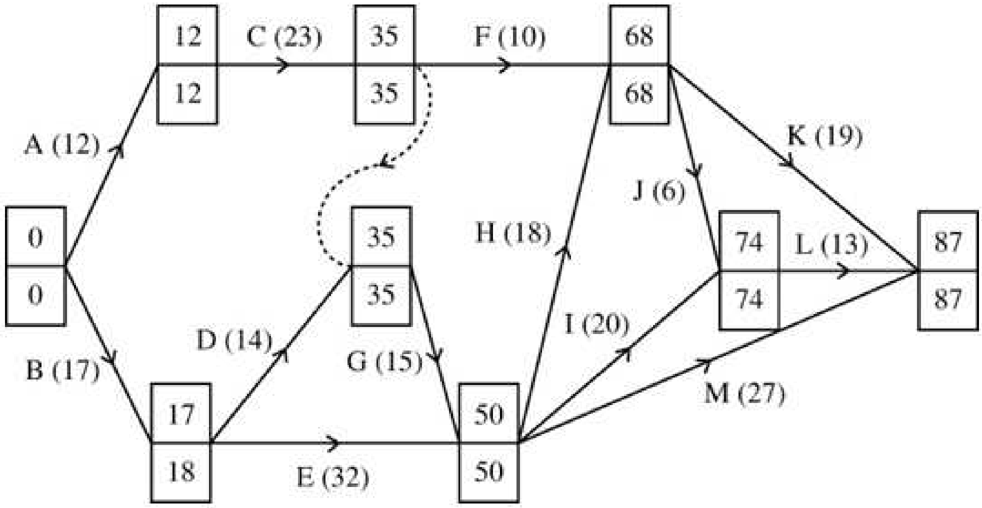
d Find a lower bound for the number of workers needed to complete the process in the minimum time.

Given that there are only three workers available, and that workers may **not** share an activity,

e schedule the activities so that the process is completed in the shortest time, use a time line. State the new shortest time. *E*

Solution:

a



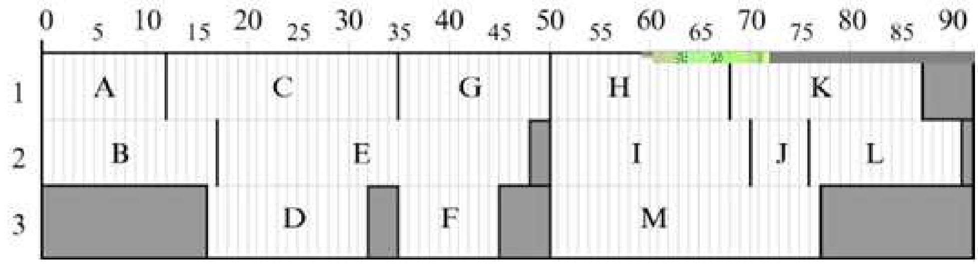
b A, C, E, H, J, K and L

All critical activities have a zero total float.

c Total float $= 35 - 17 - 14 = 4$

d *Either* $226 \div 87 = 2.6$ (1 d.p.) so at least 3 workers needed
or 69 hours into the project activities J, K, I and M *must* be happening so at least 4 workers will be needed.

e



New shortest time is 89 hours

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Review Exercise 2
Exercise A, Question 17

Question:

The Young Enterprise Company 'Decide', is going to produce badges to sell to decision maths students. It will produce two types of badge.

Badge 1 reads 'I made the decision to do maths' and
Badge 2 reads 'Maths is the right decision'

'Decide' must produce at least 200 badges and has enough material for 500 badges. Market research suggests that the number produced of Badge 1 should be between 20% and 40% of the total number of badges made.

The company makes a profit of 30p on each Badge 1 sold and 40p on each Badge 2. It will sell all that it produces, and wishes to maximise its profit.
Let x be the number produced of Badge 1 and y be the number of Badge 2.

a Formulate this situation as a linear programming problem, simplifying your inequalities so that all the coefficients are integers.

b On suitable axes, construct and clearly label the feasible region.

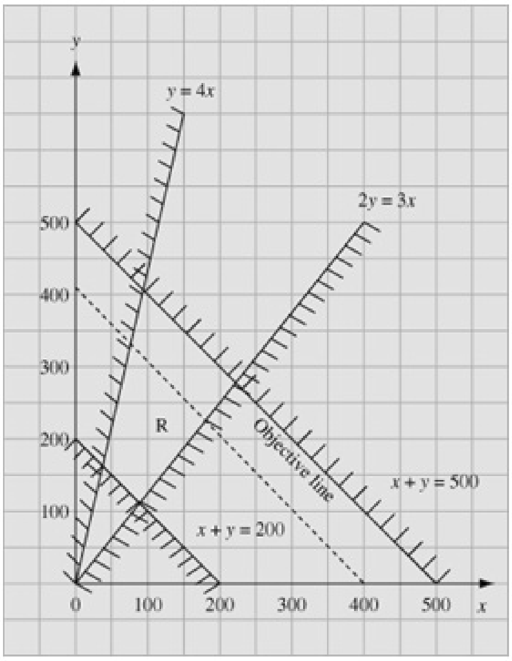
c Using your graph, advise the company on the numbers of each badge it should produce. State the maximum profit 'Decide' will make. ***E***

Solution:

a Objective: maximise $P = 30x + 40y$ (or $P = 0.3x + 0.4y$)
subject to:

$$x + y \geq 200$$
$$x + y \leq 500$$
$$x \geq \frac{20}{100}(x + y) \Rightarrow 4x \geq y$$
$$x \leq \frac{40}{100}(x + y) \Rightarrow 3x \leq 2y$$
$$x, y \geq 0$$

b



c Visible use of objective line method − objective line drawn or vertex testing − all 4 vertices tested

Vertex testing

$(40, 160) \to 7600$

$(80, 120) \to 7200$

$(100, 400) \to 19\,000$

$(200, 300) \to 18\,000$

Intersection of $y = 4x$ and $x + y = 500$
(100, 400) profit £190 (or 19 000 p)

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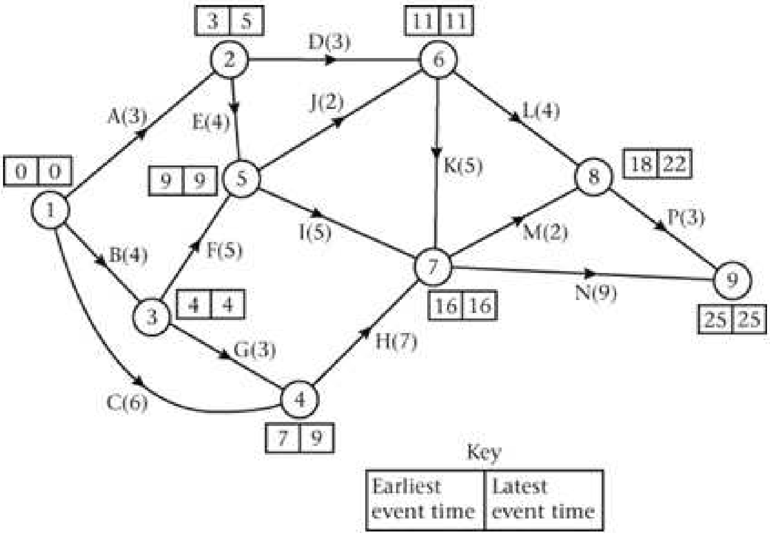
Review Exercise 2
Exercise A, Question 18

Question:

A building project is modelled by the activity network shown. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, taken to complete the activity. The left box entry at each vertex is the earliest event time and the right box entry is the latest event time



a Determine the critical activities and state the length of the critical path.

b State the total float for each non-critical activity.

c On a grid, draw a cascade (Gantt) chart for the project.

Given that each activity requires one worker,

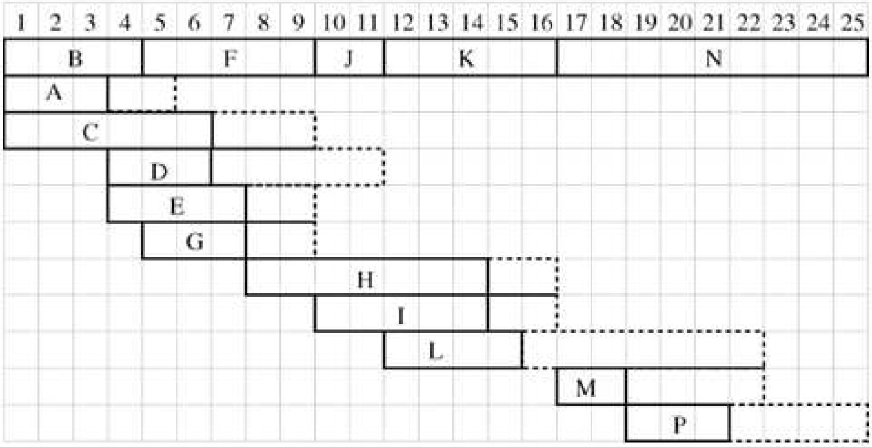
d draw up a schedule to determine the minimum number of workers required to complete the project in the critical time. State the minimum number of workers. ***E***

Solution:

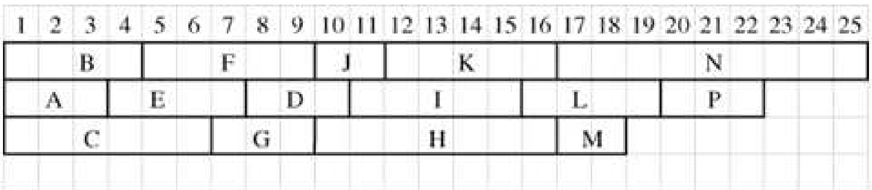
a Critical activities are B, F, J, K and N length of critical path is 25 hours
I is not critical.

b Total float on $A = 5 - 0 - 3 = 2$
Total float on $C = 9 - 0 - 6 = 3$
Total float on $D = 11 - 3 - 3 = 5$
Total float on $E = 9 - 3 - 4 = 2$
Total float on $G = 9 - 4 - 3 = 2$
Total float on $H = 16 - 7 - 7 = 2$
Total float on $I = 16 - 9 - 5 = 2$
Total float on $L = 22 - 11 - 4 = 7$
Total float on $M = 22 - 16 - 2 = 4$
Total float on $P = 25 - 18 - 3 = 4$

c



d For example



Minimum number of workers is 3.

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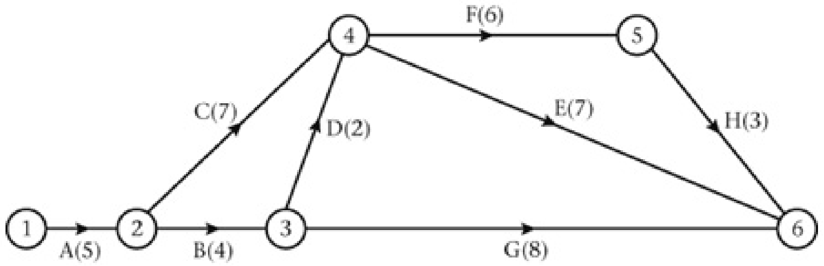
Review Exercise 2
Exercise A, Question 19

Question:



A project is modelled by the activity network shown above. The activities are represented by the edges. The number in brackets on each edge gives the time, in days, taken to complete the activity.

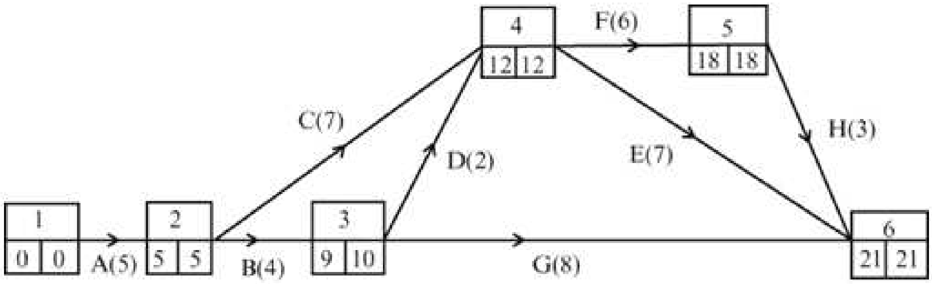
a Calculate the early time and the late time for each event. Write these in the boxes on the answer sheet.
b Hence determine the critical activities and the length of the critical path.
c Obtain the total float for each of the non-critical activities.
d Draw a cascade (Gantt) chart showing the information obtained in parts **b** and **c**.

Each activity requires one worker. Only two workers are available.

e Draw up a schedule and find the minimum time in which the 2 workers can complete the project. ***E***

Solution:

a

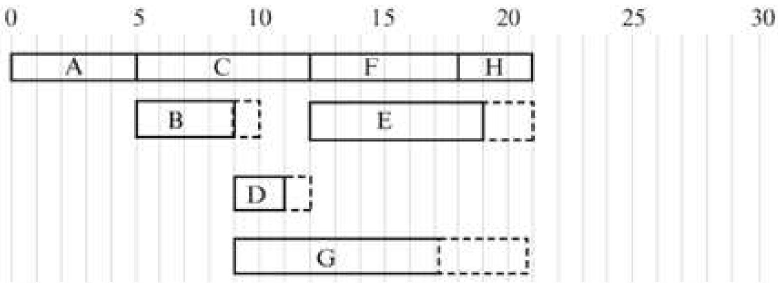


b Critical activities: A, C, F and H; length of critical path = 21

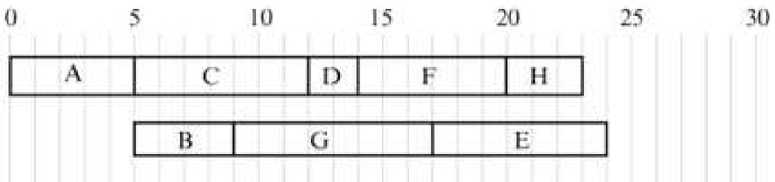
c Total float on B $= 10 - 5 - 4 = 1$ Total float on E $= 21 - 12 - 7 = 2$

Total float on D $= 12 - 9 - 2 = 1$ Total float on G $= 21 - 9 - 8 = 4$

d



e For example



Minimum time for 2 workers is 24 days.

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Review Exercise 2
Exercise A, Question 20

Question:

Becky's bird food company makes two types of bird food. One type is for bird feeders and the other for bird tables. Let x represent the quantity of food made for bird feeders and y represent the quantity of food made for bird tables. Due to restrictions in the production process, and known demand, the following constraints apply.

$$x + y \le 12,$$
$$y < 2x,$$
$$2y \ge 7,$$
$$y + 3x \ge 15.$$

a Show these constraints on a diagram and label the feasible region R.

The objective is to minimise $C = 2x + 5y$.

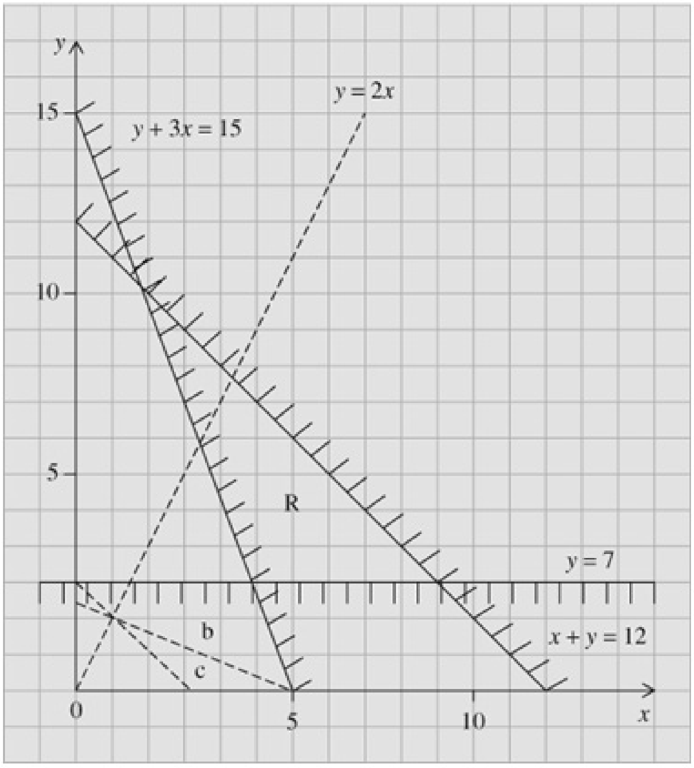
b Solve this problem, making your method clear. Give, as fractions, the value of C and the amount of each type of food that should be produced.

Another objective (for the same constraints given above) is to maximise $P = 3x + 2y$, where the variables must take integer values.

c Solve this problem, making your method clear. State the value of P and the amount of each type of food that should be produced. ***E***

Solution:

a



b Visible use of objective line method – objective line drawn or vertex testing.

$\left[\left(3\frac{5}{6}, 3\frac{1}{2}\right) \to 25\frac{1}{6} \left(8\frac{1}{2}, 3\frac{1}{2}\right) \to 34\frac{1}{2} (4, 8) \to 48\ (3, 6) \to 36\right]$

Optimal point $\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ with value $25\frac{1}{6}$

c Visible use of objective line method – objective line drawn, or vertex testing – all 4 vertices tested.

$\left(3\frac{5}{6}, 3\frac{1}{2}\right)$ not an integer try $(4,4) \to 20 \quad (4,8) \to 28$

$\left(8\frac{1}{2}, 3\frac{1}{2}\right)$ not an integer try $(8,4) \to 32 \quad (3,6) \to 21$

Optimal point (8, 4) with value 32

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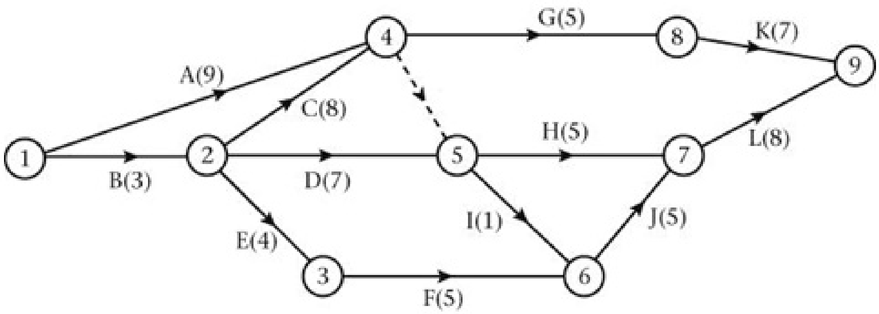
Review Exercise 2
Exercise A, Question 21

Question:



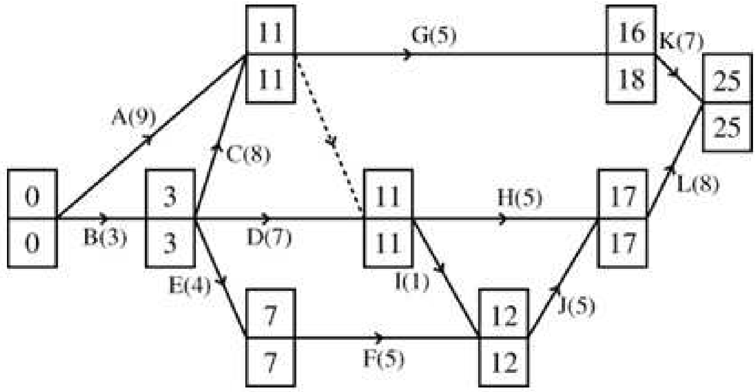
A project is modelled by the activity network shown above. The activities are represented by the arcs. The number in brackets on each arc gives the time, in hours, to complete the activity. The numbers in circles give the event numbers. Each activity requires one worker.

a Explain the purpose of the dotted line from event 4 to event 5.
b Calculate the early time and the late time for each event.
c Determine the critical activities.
d Obtain the total float for each of the non-critical activities.
e On a grid, draw a cascade (Gantt) chart, showing the answers to parts **c** and **d**.
f Determine the minimum number of workers needed to complete the project in the minimum time. Make your reasoning clear. ***E***

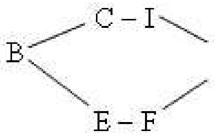
Solution:

a For example, it shows dependence but it is not an activity. G depends on A and C *only* but H and I depend on A, C *and* D.

b



c



So B, C, E, F, I, J and L

d Total float on $A = 11 - 0 - 9 = 2$

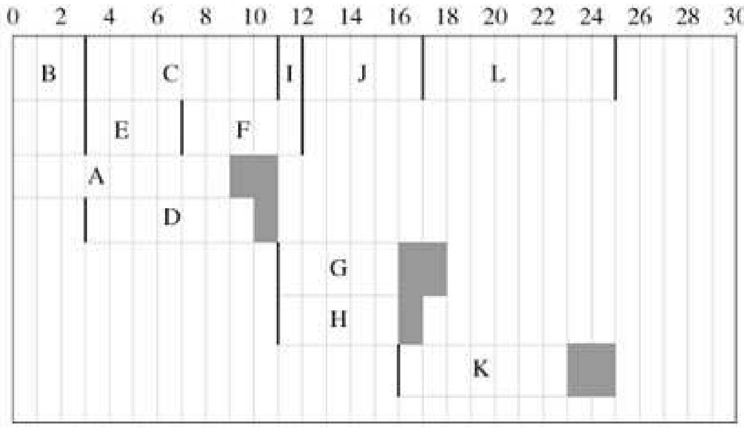
Total float on $D = 11 - 3 - 7 = 1$

Total float on $G = 18 - 11 - 5 = 2$

Total float on $H = 17 - 11 - 5 = 1$

Total float on $K = 25 - 16 - 7 = 2$

e



f Arithmetic lower bound $= \dfrac{67}{25} = 2.68$ so a minimum of 3 workers needed.

From Gantt chart: At time 8 activities C, F, A and D must be happening, so a minimum of 4 workers are needed.

We need to take the higher of these as our best lower bound and state that a minimum of 4 workers are needed.

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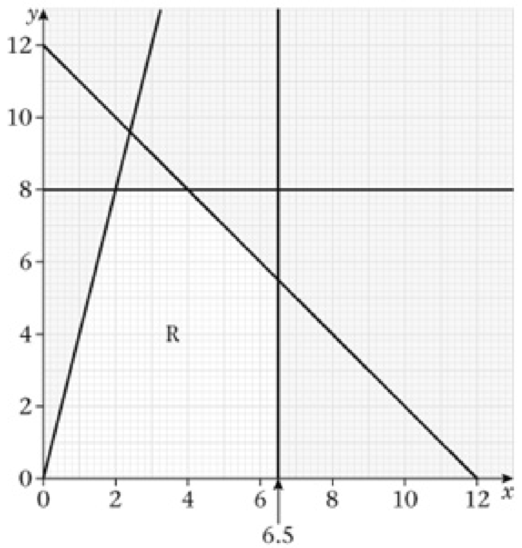
Review Exercise 2
Exercise A, Question 22

Question:

The company EXYCEL makes two types of battery, X and Y. Machinery, workforce and predicted sales determine the number of batteries EXYCEL make. The company decides to use a graphical method to find its optimal daily production of X and Y.

The constraints are modelled in the diagram. The feasible region, R, is indicated.



x = the number (in thousands) of type X batteries produced each day,
y = the number (in thousands) of type Y batteries produced each day.
The profit of each type X battery is 40p and on each type Y battery is 20p.
The company wishes to maximise its daily profit.

a Write this as a linear programming problem, in terms of x and y, stating the objective function and all the constraints.
b Find the optimal number of batteries to be made each day. Show your method clearly.
c Find the daily profit, in £, made by EXYCEL. ***E***

Solution:

a Objective: maximise $P = 0.4x + 0.2y \quad (P = 40x + 20y)$

subject to:

$$x \le 6.5$$
$$y \le 8$$
$$x + y \le 12$$
$$y \le 4x$$
$$x, y \ge 0$$

b Visible use of objective line method – objective line drawn (e.g. from (2, 0) to (0, 4)) or all 5 points tested.

vertex testing

$[(0,0) \to 0; (2,8) \to 2.4; (4,8) \to 3.2; (6.5,5.5) \to 3.7; (6.5,0) \to 2.6]$

Optimal point is $(6.5, 5.5) \Rightarrow 6\,500$ type X and 5500 type Y

c $P = 0.4(6500) + 0.2(5500) = £3700$

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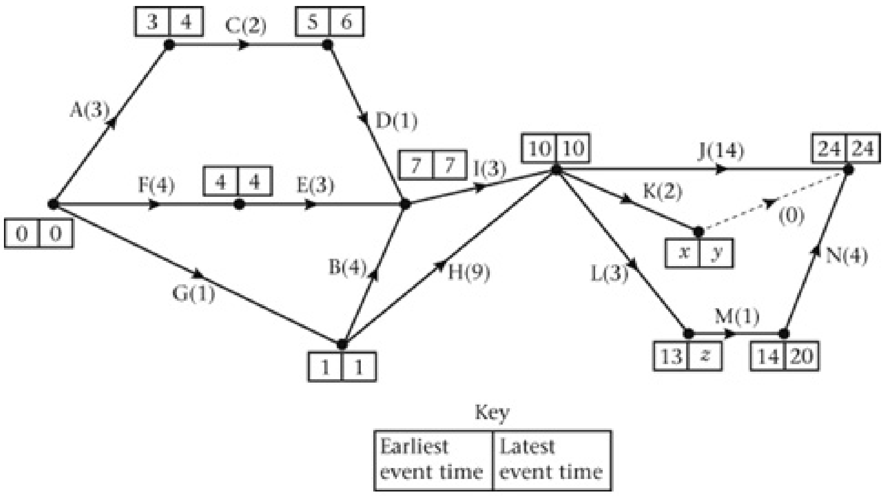
Review Exercise 2
Exercise A, Question 23

Question:



The network above shows activities that need to be undertaken in order to complete a project. Each activity is represented by an arc. The number in brackets is the duration of the activity in hours. The early and late event times are shown at each node. The project can be completed in 24 hours.

a Find the values of x, y and z.

b Explain the use of the dummy activity in the diagram.

c List the critical activities.

d Explain what effect a delay of one hour to activity B would have on the time taken to complete the whole project.

The company which is to undertake this project has only two full time workers available. The project must be completed in 24 hours and in order to achieve this, the company is prepared to hire additional workers at a cost of £28 per hour. The company wishes to minimise the money spent on additional workers. Any worker can undertake any task and each task requires only one worker.

e Explain why the company will have to hire additional workers in order to complete the project in 24 hours.

f Schedule the tasks to workers so that the project is completed in 24 hours and at minimum cost to the company.

g State the minimum extra cost to the company. ***E***

Solution:

a $x = 12 \quad y = 24 \quad z = 19$

b Allows J and K to be uniquely expressed in terms of their end events.

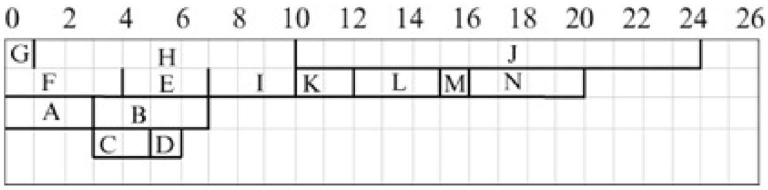
c F, E, I, J, G, H

d It would have no effect, B has a total float of 2 so a delay of one hour would still permit the project to be completed on time.

e For example,

- the total of activities is 54. 2 workers working for 24 hours would not be sufficient,
- $54 \div 24 = 2.25$, so 2 workers are not enough,
- 7 hours into the project A, B, C, D, E, F and G must be completed; these activities require 18 hours to complete them so 2 workers could not be enough.

f



g 10 extra hours $(7+3)$ so £280

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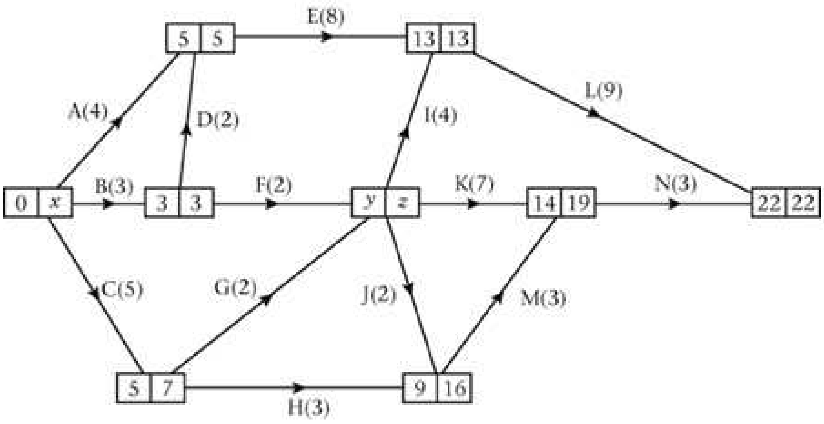
Review Exercise 2
Exercise A, Question 24

Question:



A trainee at a building company is using critical path analysis to help plan a project. The diagram shows the trainee's activity network. Each activity is represented by an arc and the number in brackets on each arc is the duration of the activity, in hours.

a Find the values of x, y and z.

b State the total length of the project and list the critical activities.

c Calculate the total float time on

  i activity N,

  ii activity H.

The trainee's activity network is checked by the supervisor who finds a number of errors and omissions in the diagram. The project should be represented by the following precedence table.

Activity	Must be preceded by:	Duration
A	–	4
B	–	3
C	–	5
D	B	2
E	A, D	8
F	B	2
G	C	2
H	C	3
I	F, G	4
J	F, G	2
K	F, G	7
L	E, I	9
M	H, J	3
N	E, I, K, M	3
P	E, I	6
Q	H, J	5
R	Q	7

d By adding activities and dummies amend the diagram so that it represents this precedence table.

e Find the total time needed to complete this project. ***E***

Solution:

a $x = 0$

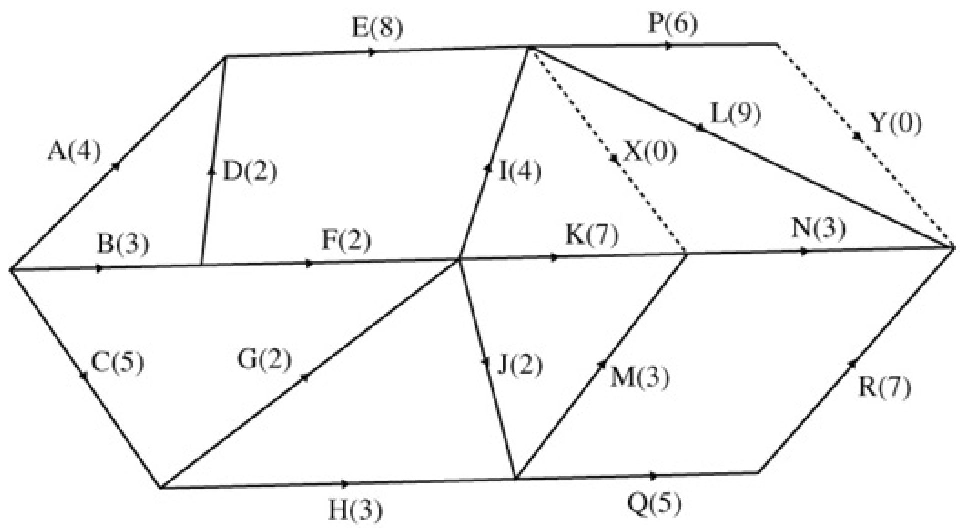
$y = 7$ [latest out of $(3+2)$ and $(5+2)$]

$z = 9$ [Earliest out of $(13-4)$ and $(19-7)$ and $(16-2)$]

b Length is 22
Critical activities: B, D, E and L

c Total float on $\text{N} = 22 - 14 - 3 = 5$
Total float on $\text{H} = 16 - 5 - 3 = 8$

d



e 22 hours [critical path is still B–D–E–L]

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Review Exercise 2
Exercise A, Question 25

Question:

A chemical company produces two products X and Y. Based on potential demand, the total production each week must be at least 380 gallons. A major customer's weekly order for 125 gallons of Y must be satisfied.

Product X requires 2 hours of processing time for each gallon and product Y requires 4 hours of processing time for each gallon. There are 1200 hours of processing time available each week. Let x be the number of gallons of X produced and y be the number of gallons of Y produced each week.

a Write down the inequalities which x and y must satisfy.

It costs £3 to produce 1 gallon of X and £2 to produce 1 gallon of Y. Given that the total cost of production is £C,

b express C in terms of x and y.

The company wishes to minimise the total cost.

c Using a graphical method, solve the resulting linear programming problem. Find the optimal values of x and y and the resulting total cost.

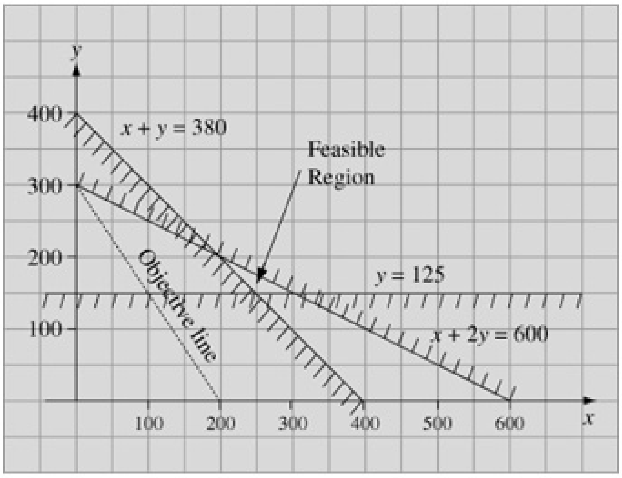
d Find the maximum cost of production for all possible choices of x and y which satisfy the inequalities you wrote down in part **a**. ***E***

Solution:

a $x+y \geq 380$ (total production is at least 380)
$y \geq 125$ (at least 125 gallons of y)
$2x+4y \leq 1200 \Rightarrow x+2y \leq 600$ (processing time)
$x \geq 0$

b $C = 3x+2y$

c



Visible use of objective line method − objective line drawn or vertex testing − all 3 vertices $[(160,220) \to 920; (255,125) \to 1015; (350,125) \to 1300]$
Optimal point is (160, 220) value £920.

d (350, 125) $C = £1300$

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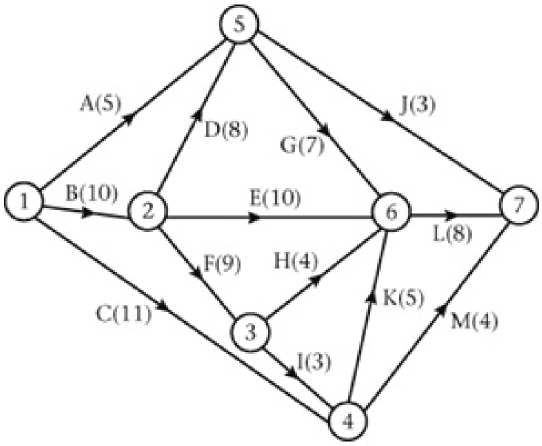
Review Exercise 2
Exercise A, Question 26

Question:



The diagram shows the activity network used to model a building project. The activities are represented by the edges. The number in brackets on each edge represents the time, in days, to complete the activity.

a Calculate the early time and the late time for each event.

b Calculate the total float for each activity.

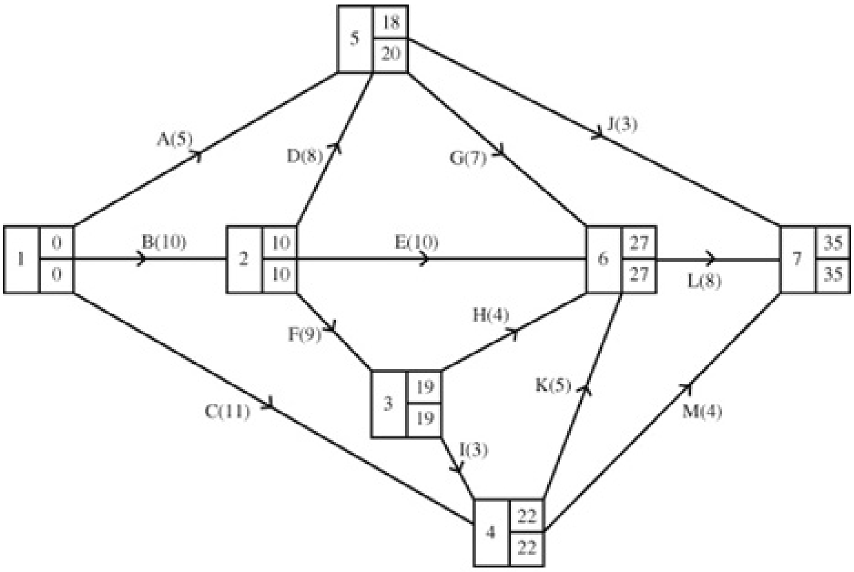
c Hence determine the critical activities. Write down the length of the critical path.

Owing to the breakdown of a piece of equipment the time taken to complete activity H increases to 9 days.

d Obtain the new critical path and its length. ***E***

Solution:

a



b Total float on $A = 20 - 0 - 5 = 15$
Total float on $B = 10 - 0 - 10 = 0$
Total float on $C = 22 - 0 - 11 = 11$
Total float on $D = 20 - 10 - 8 = 2$
Total float on $E = 27 - 10 - 10 = 7$
Total float on $F = 19 - 10 - 9 = 0$
Total float on $G = 27 - 18 - 7 = 2$
Total float on $H = 27 - 19 - 4 = 4$
Total float on $I = 22 - 19 - 3 = 0$
Total float on $J = 35 - 18 - 3 = 14$
Total float on $K = 27 - 22 - 5 = 0$
Total float on $L = 35 - 27 - 8 = 0$
Total float on $M = 35 - 22 - 4 = 9$

c Critical activities: B, F, I, K and L
length of critical path is 35 days

d New critical path is B–F–H–L
length of new critical path is 36 days